Lecture 22

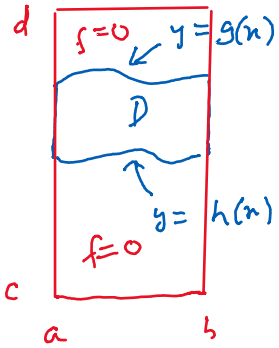
Monday, June 3, 2024 12:57 PM

$$\iint_D f(x,y)\,dA = \lim_{\substack{m\to\infty \\ n\to\infty}} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_i,y_j)\Delta x\Delta y \quad : \text{double integral}$$

$$\iint_{[a,b]\times[c,d]} f(x,y)\,dA = \int_a^b\int_c^d f(x,y)\,dy\,dx = \int_c^d\int_a^b f(x,y)\,dx\,dy$$

iterated integral

What if D is not a rectangle?



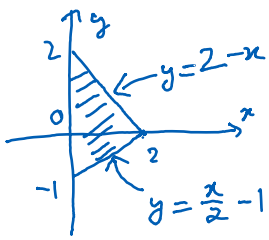
$$D = \{(x,y) : a\le x\le b,\ h(x)\le y\le g(x)\}$$

$$\iint_D f(x,y)\,dA = \int_a^b\int_c^d f(x,y)\,dy\,dx$$

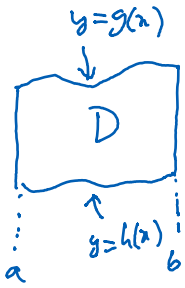
$$= \int_a^b \left(\underbrace{\int_c^{h(x)}}_{0} + \int_{h(x)}^{g(x)} + \underbrace{\int_{g(x)}^{d}}_{0} \right) f(x,y)\,dy\,dx$$

$$= \int_a^b \int_{h(x)}^{g(x)} f(x,y)\,dy\,dx$$

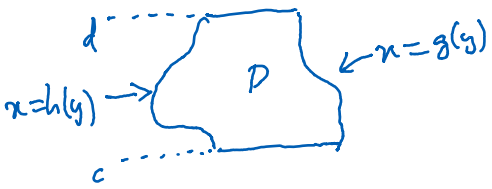
Ex



$$\iint_D y\,dA = \int_0^2\int_{\frac{x}{2}-1}^{2-x} y\,dy\,dx = \dots$$

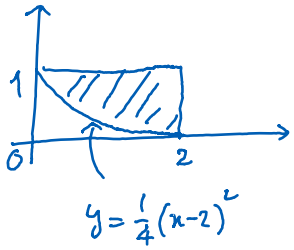


$$\iint_D f(x,y)\,dA = \int_a^b \int_{h(x)}^{g(x)} f(x,y)\,dy\,dx$$



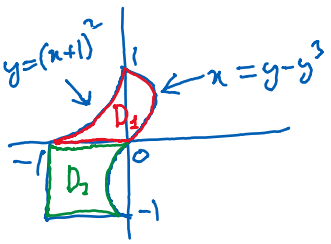
$$\iint_D f(x,y)\,dA = \int_c^d \int_{h(y)}^{g(y)} f(x,y)\,dx\,dy$$

Ex



$$\iint_D x\,dA = \text{using both methods}$$

Ex



$$\iint_D xy\,dA = \iint_{R_1} xy\,dA + \iint_{R_2} xy\,dA$$

$$= \int_0^1 \int_{\sqrt{y}-1}^{y-y^3} xy\,dx\,dy + \int_{-1}^{1} \int_{-1}^{y-y^3} xy\,dx\,dy$$

$$= \ldots$$

Use Mathematica to evaluate double / iterated integrals.