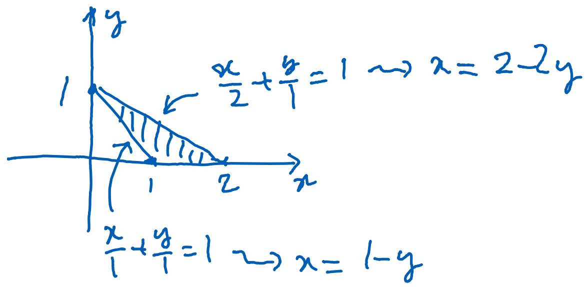
Lecture 23

Tuesday, June 4, 2024 9:20 AM

More practice with double integral over a general domain:

Ex Find the volume of the solid under the surface $z = xy$ and above the triangle with vertices $(0,1), (1,0), (2,0)$ on the xy-plane.

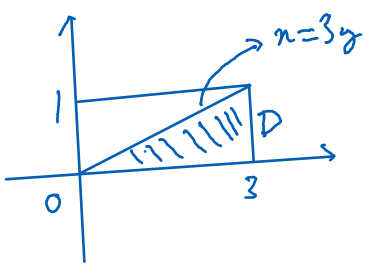


$$\text{vol} = \iint_D xy\,dA$$

$$= \int_0^1 \int_{1-y}^{2-2y} xy\,dx\,dy = \dots$$

Sometimes, we need to change the order of integration.

Ex Evaluate $\underbrace{\int_0^1 \int_{3y}^{3} e^{x^2}\,dx\,dy}_{\text{difficult}} = \iint_D e^{x^2}\,dA$



$$D = \{(x,y) : 0 \le y \le 1,\ 3y \le x \le 3\}$$

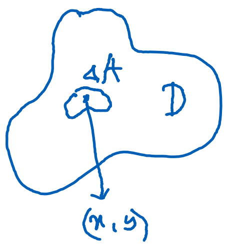
$$\iint_D e^{x^2}\,dA = \int_0^3 \int_0^{x/3} e^{x^2}\,dy\,dx$$

$$= \int_0^3 \frac{x}{3} e^{x^2}\,dx$$

Let $u = x^2$, $du = u'dx = 2x\,dx$, $dx = \frac{du}{2x}$

$$\int_0^3 \frac{x}{3} e^{x^2}\,dx = \int_0^9 \frac{x}{3} e^u \frac{du}{2x} = \frac{1}{6}\int_0^9 e^u\,du = \frac{e^9 - 1}{6}$$

* Physical application of double integral:



D: metal plate with uneven mass distribution.

$f(x,y)$ = mass density

mass of $\Delta A \approx f(x,y)\Delta A$

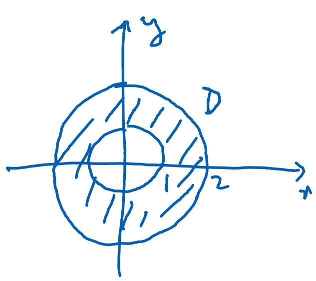
total mass $= \sum f(x,y)\Delta A \approx \iint_D f(x,y)\,dA$

Special case: $f(x,y) = C$ (constant)

$$\iint_D f(x,y)\,dA = C \cdot \text{area}(D)$$

If $C=1$: $\displaystyle\iint_D 1\,dA = \text{area}(D)$

What was impossible in Calc II may be possible now by changing the order of integration (going into the house by the back door instead of the front door which is locked).



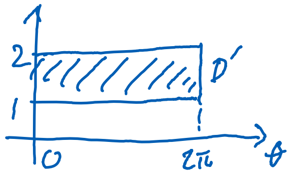
$$\iint_D (x^2+y^2)\,dA = ?$$

D is a tricky region, not in type 1 or type 2.

$D = \{(x,y) : 1 \le \sqrt{x^2+y^2} \le 2\}$

Polar coords: $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ $\quad 1 \le r \le 2,\ 0 \le \theta \le 2\pi$

$D \longleftrightarrow D' = \{(r,\theta) : 0 \le r \le 1,\ 0 \le \theta \le 2\pi\}$



D is "transformed" into a nicer region $D' = [1,2] \times [0,2\pi]$.