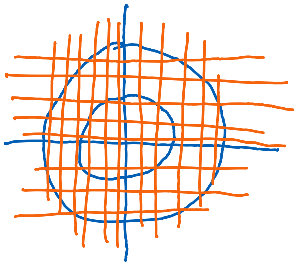
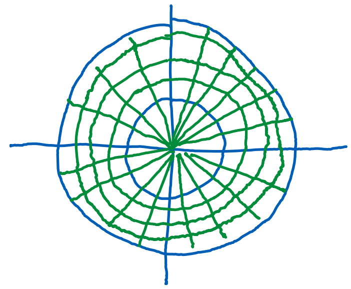
Lecture 24

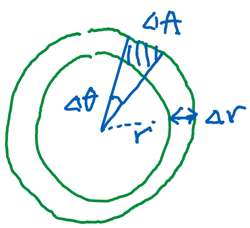
Wednesday, June 5, 2024 8:43 AM





Two ways of partitioning the annulus.

$$\iint_D f(x,y)\,dy \approx \sum f(x_{ij}, y_{ij})\,\Delta A_{ij}$$



The area of the annulus between r and $r+\Delta r$ is

$$\pi(r+\Delta r)^2 - \pi r^2 = 2\pi r\Delta r + \underbrace{\pi\Delta r^2}_{\text{small of order 2}} \approx 2\pi r\Delta r.$$

The area of the piece ΔA is $\frac{\Delta\theta}{2\pi}\, 2\pi r\Delta r = r\Delta r\Delta\theta$.

Thus, $\Delta A = r\Delta r\Delta\theta$.

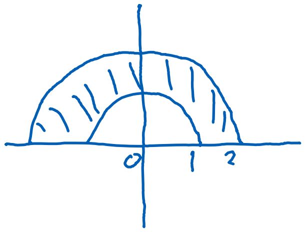
$$\iint_D f(x,y)\,dA \approx \sum f(x_{ij}, y_{ij})\,\Delta A_{ij} = \sum \bar{f}(r_{ij}, \theta_{ij})\, r_{ij}\,\Delta r\Delta\theta$$

$$\approx \iint_{D'} \bar{f}(r,\theta)\, r\,dr\,d\theta$$

Here $\bar{f}(r,\theta) = f(x,y) = x^2+y^2 = r^2$

$$\iint_D (x^2+y^2)\,dA = \int_0^{2\pi}\int_1^2 r^2\, r\,dr\,d\theta = \int_0^{2\pi} \frac{r^4}{4}\Big|_1^2 d\theta = \int_0^{2\pi} \frac{15}{4}\,d\theta = \frac{15\pi}{2}.$$

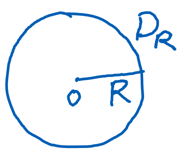
Ex



Find $\displaystyle\iint_D 3x\, dA$

Ex

$$\iint_{\mathbb{R}^2} e^{-x^2-y^2} dA = \lim_{R\to\infty} \iint_{D_R} e^{-x^2-y^2} dA = \dots \quad \text{(the result has } \pi.)$$

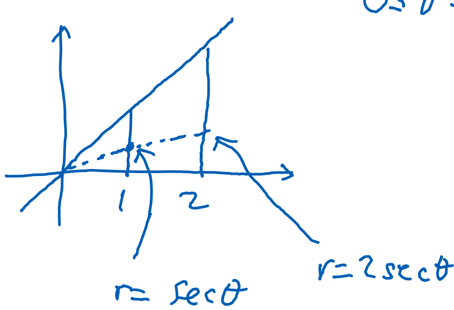


Ex

$$\int_1^2\int_0^x \frac{1}{(x^2+y^2)^{3/2}}\, dy\, dx = \iint_D \frac{1}{(x^2+y^2)^{3/2}}\, dA$$

$$0 \le \theta \le \frac{\pi}{4}, \quad \sec\theta \le r \le 2\sec\theta$$

$$(x^2+y^2)^{3/2} = r^3$$



$$\iint_D \frac{1}{(x^2+y^2)^{3/2}}\, dA = \int_0^{\pi/4}\int_{\sec\theta}^{2\sec\theta} \frac{1}{r^3}\, r\, dr\, d\theta = \int_0^{\pi/4} -\frac{1}{r}\Big|_{\sec\theta}^{2\sec\theta} d\theta$$

$$= \int_0^{\pi/4} \left(\frac{1}{\sec\theta} - \frac{1}{2\sec\theta}\right) d\theta = \int_0^{\pi/2} \frac{1}{2}\cos\theta\, d\theta$$

$$= \frac{1}{2}\sin\theta\Big|_0^{\pi/2} = \frac{1}{2}.$$