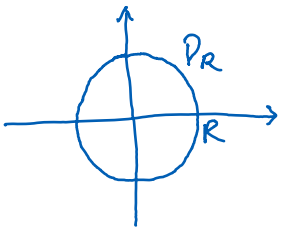
Lecture 25

Thursday, June 6, 2024 1:31 PM

Compute $\int_{-\infty}^{\infty} e^{-x^2}\,dx$ ...

$$\left(\int_{-\infty}^{\infty} e^{-x^2}\,dx\right)^2 = \int_{-\infty}^{\infty} e^{-x^2}\,dx \int_{-\infty}^{\infty} e^{-y^2}\,dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2}\,dx\,dy = \iint_{\mathbb{R}^2} e^{-x^2-y^2}\,dA$$

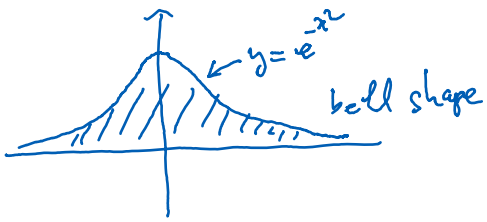
$$= \lim_{R\to\infty} \iint_{D_R} e^{-x^2-y^2}\,dA$$



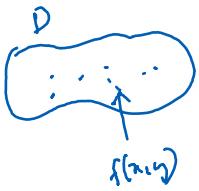
$$\iint_R e^{-x^2-y^2}\,dA = \int_0^R \int_0^{2\pi} e^{-r^2}\, r\,d\theta\,dr$$

$$= \pi\left(1-e^{-R^2}\right) \xrightarrow{R\to\infty} \pi$$

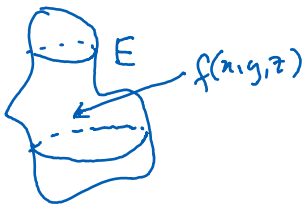
Thus, $\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$



Triple integral

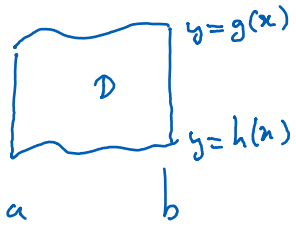


Total mass $= \iint_D f(x,y)\,dA$

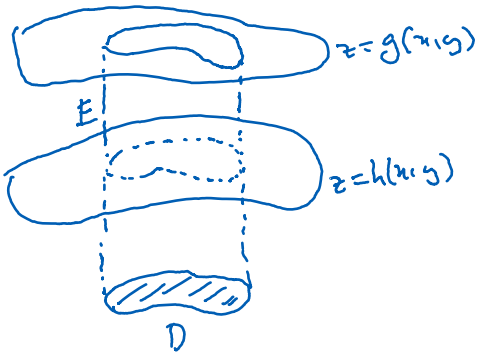


Total mass $= \iiint_E f(x,y,z)\,dV$

$$\iiint_{[a,b]\times[c,d]\times[k,l]} f(x,y,z)\,dV = \int_a^b \int_c^d \int_k^l f(x,y,z)\,dz\,dy\,dx$$



$$\iint_D f(x,y)\,dA = \int_a^b \int_{h(x)}^{g(x)} f(x,y)\,dy\,dx$$



$$E = \{(x,y,z) : (x,y) \in D,\ h(x,y) \le z \le g(x,y)\}$$

$$\iiint_E f(x,y,z)\,dV = \iint_D \int_{h(x,y)}^{g(x,y)} f(x,y,z)\,dz\,dA$$