Lecture 27

Monday, June 10, 2024 8:19 AM

Practice with drawing solids from inequalities:

1) $E=\{(x,y,z): 0\le z\le 1-y,\ \sqrt{x}\le y\le 1,\ 0\le x\le 1\}$

Sketch E and find the volume of E.

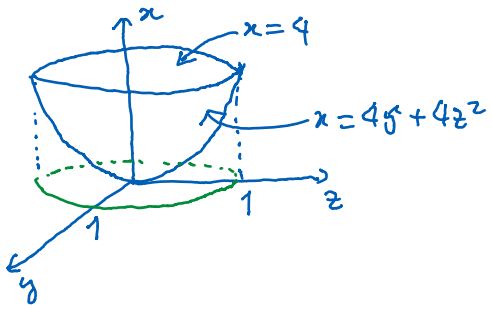
Note: Once x moves freely from 0 to 1, y can't move freely from 0 to 1. Imagine that you have \$50. You go into a store and want to buy 2 items. You have the freedom to buy the first item (anywhere between 0 and 50 dollars). But you don't have such a freedom to buy the second item. The price range of the second item depends on the price of the first item.

2) $E=\{(x,y,z): 0\le y\le 2,\ 0\le z\le 2-y,\ 0\le x\le 4-y^2\}$

Sketch E and find the volume of E

More practice with triple integral:

3) $\iiint_E x\,dV$ where E is the solid bounded by $x=4y^2+4z^2$ and $x=4$.



$D=\{(y,z): y^2+z^2\le 1\}$

$E=\{(x,y,z): (y,z)\in D,\ 4y^2+4z^2\le x\le 4\}$

$$\iiint_E x\,dV=\iint_D\int_{4y^2+4z^2}^{4} x\,dx\,dA$$

$$=\iint_D \frac{x^2}{2}\Big|_{x=4y^2+4z^2}^{x=4} dA=\iint_D \frac{16-(4y^2+4z^2)^2}{2}\,dA=8\iint_D\left[1-(y^2+z^2)^2\right]dA$$

$$= 8\int_0^1\int_0^{2\pi}(1-r^2)^2 r\,d\theta\,dr = 16\pi\int_0^1 (1-t)^2 r\,dr = \dots$$

Cylindrical coords:

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases} \qquad (x,y,z) \longrightarrow (r,\theta,z)$$

$$\iiint_E f(x,y,z)\,dV = \iiint_{\overline{E}} \bar{f}(r,\theta,z)\, r\,dr\,d\theta\,dz$$

Ex $\iiint_E z\,dV$ where E is the solid bounded by the cone $z=\sqrt{x^2+y^2}$ and the cylinder $x^2+y^2=4$.

Ex Find the volume of the solid described by the inequalities

$$r^2 \le z \le 8 - r^2$$

in cylindrical coordinates.

Ex Find the volume of the solid obtained by revolving the region $D = \{(x,y) : 0 \le y \le \sqrt{x},\ 0 \le x \le 4\}$ about the x-axis.