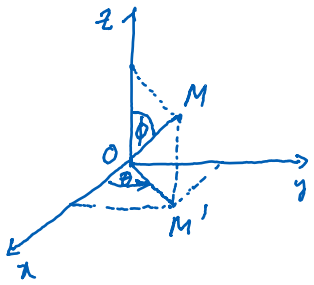
Lecture 29

Wednesday, June 12, 2024 1:55 PM



$z = \rho\cos\phi$

$x = OM'\cos\theta = \rho\sin\phi\cos\theta$

$y = OM'\sin\theta = \rho\sin\phi\sin\theta$

$\rho \geq 0$

$0 \leq \phi \leq \pi$

$0 \leq \theta \leq 2\pi$

Ex Convert $(\rho, \phi, \theta) = (2, \frac{\pi}{4}, \frac{\pi}{6})$ to (x, y, z)

Ex Convert $(x, y, z) = (1, \sqrt{2}, -1)$ to (ρ, ϕ, θ)

Triple integral using spherical coordinates:

$$\iiint_E f(x,y,z)\,dV = \iiint_{E'} \tilde{f}(\rho,\theta,\phi)\,\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$$

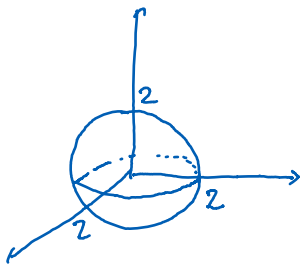
order "negotiable"

$(x, y, z) \in E$: old solid

$(\rho, \theta, \phi) \in E'$: new solid

Ex: $\iiint_E (x^2+y^2+z^2)^2\,dV$

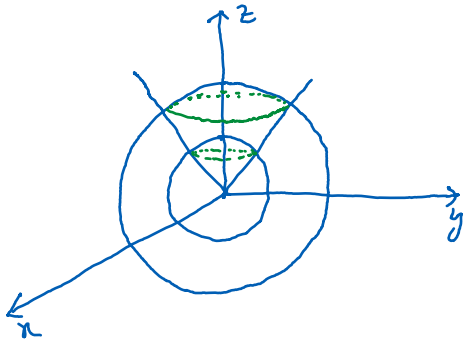
where E is the ball of radius 2 centered at the origin.



$$\left.\begin{array}{l} 0 \leq \rho \leq 2 \\ 0 \leq \phi \leq \pi \\ 0 \leq \theta \leq 2\pi \end{array}\right\} E = [0,2] \times [0,2\pi] \times [0,\pi]$$

$$\iiint_E \ldots dV = \int_0^2 \int_0^{2\pi} \int_0^{\pi} \rho^4 \rho^2 \sin\phi\,d\phi\,d\theta\,d\rho$$

Ex $\iiint_E z\,dV$ where E is the solid bounded by the cone $z=\sqrt{x^2+y^2}$, the sphere $x^2+y^2+z^2=1$ and the sphere $x^2+y^2+z^2=4$.



$1 \le \rho \le 2$

$0 \le \theta \le 2\pi$

$0 \le \phi \le \frac{\pi}{4}$

Finish this problem for extra credit.