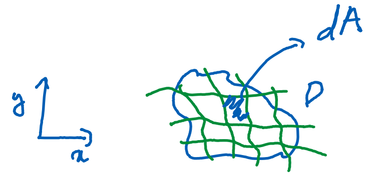
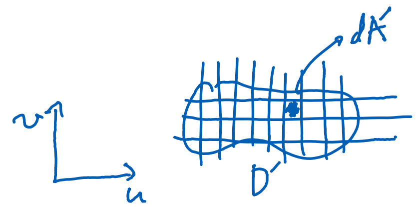
Lecture 30

Thursday, June 13, 2024 1:18 PM

* Change of variables





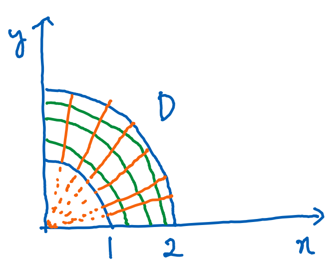
$$\frac{dA}{dA'} = \left|\frac{\partial(x,y)}{\partial(u,v)}\right|$$ This is the stretching ratio

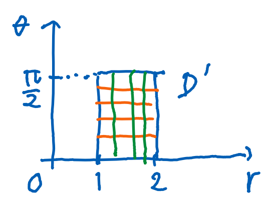
where $$\frac{\partial(x,y)}{\partial(u,v)} = \det \underbrace{\begin{pmatrix} x_u & x_v \\ y_u & y_v \end{pmatrix}}_{\text{Jacobian matrix}}$$

$$\iint_D f(x,y)\, dA = \iint_{D'} \bar{f}(u,v) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| dA'$$

Ex $(x,y) \to (r,\theta)$

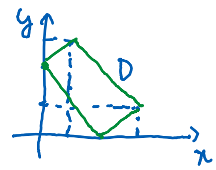
$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \qquad \frac{\partial(x,y)}{\partial(u,v)} = \det\begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix} = r(\cos^2\theta + \sin^2\theta) = r$$

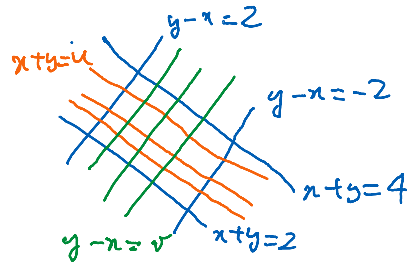


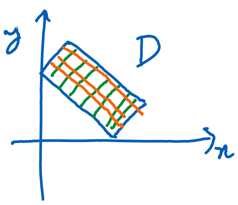


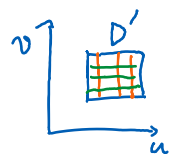
Ex Evaluate $\iint_D (x+y)\,dA$

where D is the parallelogram with vertices at $(2,0)$, $(0,2)$, $(1,3)$, $(3,1)$.









$D' = [2,4] \times [-2,2]$.

$$J = \left| \frac{\partial(x,y)}{\partial(u,v)} \right| = \left| \det \begin{pmatrix} x_u & x_v \\ y_u & y_v \end{pmatrix} \right|$$

Note that we can solve for x, y in terms of u, v:

$$x = \frac{u-v}{2}, \quad y = \frac{u+v}{2}$$

$$J = \left| \det \begin{pmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{pmatrix} \right| = \frac{1}{2}$$

$$\iint_D (x+y)\,dA = \iint_{D'} u \, \frac{1}{2}\, du\,dv = \int_{-2}^{2}\int_{2}^{4} \frac{u}{2}\, du\,dv = 12$$