Lecture 32

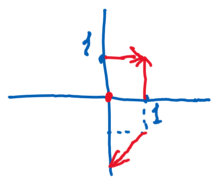
Monday, June 17, 2024 5:30 AM

* Vector fields:

$$F: \underbrace{D \subset \mathbb{R}^2}_{\text{points}} \to \underbrace{\mathbb{R}^2}_{\text{vectors}}$$

Ex $F(x,y) = (y,x)$

(x,y)	$F(x,y)$
$(0,0)$	$(0,0)$
$(1,0)$	$(0,1)$
$(0,1)$	$(1,0)$
$(1,-1)$	$(-1,1)$



Use Mathematica to plot a vector field:

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VectorPlot[{y, x}, {x, -5, 5}, {y, -5, 5}]

VectorPlot3D[{y, -z, x}, {x, -5, 5}, {y, -5, 5}, {z, -5, 5}]
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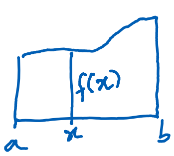
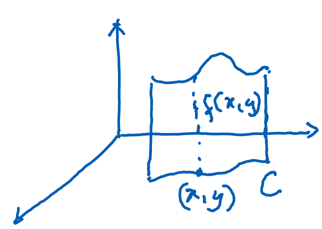
Line integral

Calc II integral
- → double/triple integral
- → line integral
  - → scalar function
  - → vector field

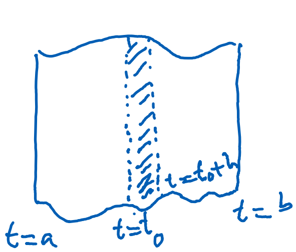


area $= \int_a^b f(x)\,dx$



area $= \int_C f(x,y)\,ds$

Let $r(t) = (x(t), y(t))$ be a parametrization of C.



$A(t) =$ wall area up from a to t.

$A(t_0+h) - A(t_0) \approx$ area of rectangle with width $|r(t_0+h) - r(t_0)|$ and height $f(x(t_0), y(t_0))$.

$$= |r(t_0+h) - r(t_0)|\, f(x(t_0), y(t_0)).$$

Thus,

$$\frac{A(t_0+h) - A(t_0)}{h} \approx \left| \frac{r(t_0+h) - r(t_0)}{h} \right| f(x(t_0), y(t_0))$$

Let $h \to \infty$:

$$A'(t_0) = |r'(t_0)|\, f(x(t_0), y(t_0))$$

Drop the subscript:

$$A'(t) = |r'(t)|\, f(x(t), y(t))$$

Thus,

$$A(t) = \underbrace{A(a)}_{=0} + \int_a^b A'(t)\, dt = \int_a^b f(x(t), y(t))\, |r'(t)|\, dt$$

<u>Definition</u>:

$$\int_C f(x,y)\, ds = \int_a^b f(x(t), y(t))\, |r'(t)|\, dt$$