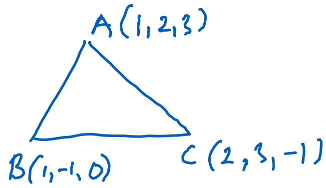
Lecture 4

Monday, May 6, 2024 8:55 AM

Practice with dot and cross product:



Find $\vec{AB}\cdot(\vec{AC}\times\vec{BC})$ and $\vec{AB}\times(\vec{AC}\times\vec{BC})$

Note:

$v\times w = -w\times v$

$v\times(u+w) = v\times u + v\times w$

$v\times(2w) = 2\,v\times w$

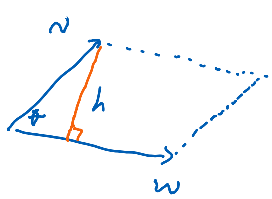
$v\times(u\times w) \neq (v\times u)\times w$ (non-associative)

Geometric meaning of $v\times w$.

$v\times w$ is a vector, so it has a length and a direction.

The direction is determined by the right-hand rule (or the screwdriver rule)

The length is the area of the parallelogram formed by v and w



area $= h|w| = |v|\sin\theta\,|w| = |v||w|\sin\theta$

Therefore, $|v\times w| = |v||w|\sin\theta$.

Recall: $v\cdot w = |v||w|\cos\theta$.

Ex: Find the area of the triangle formed by

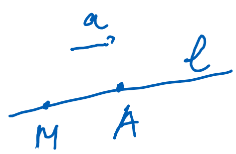
$A(1,2,3)$, $B(1,-1,0)$, $C(2,3,-1)$

Ex Find the area of the triangle formed by

$A(1,2)$, $B(1,-1)$, $C(2,3)$

Note: the 3D vectors u, v, w are called coplanar if they can be fit in the same plane. Equivalently, u, v, w are coplanar if and only if $\underbrace{u \cdot (v \times w)}_{\text{triple product}} = 0$

* Cross product and dot product are used to described planes and lines.



$A(x_0, y_0, z_0)$

$a = (a_1, a_2, a_3)$

$\overrightarrow{AM} = ta \rightsquigarrow (x - x_0, y - y_0, z - z_0) = (ta_1, ta_2, ta_3)$

$$\rightsquigarrow \begin{cases} x = x_0 + ta_1 \\ y = y_0 + ta_2 \\ z = z_0 + ta_3 \end{cases}$$

This is called parametric equations of the line ℓ.