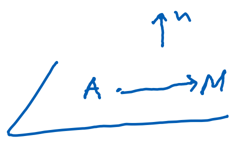
Lecture 5

Tuesday, May 7, 2024 8:50 AM

Lines and planes

One can describe a plane using 3 points on the plane. This method requires 9 data.

Can we describe a plane using less data?



Suppose we know a point A on the plane and a normal vector of the plane. All points M on the plane must satisfy $\overrightarrow{AM} \perp n$. In other words,

$$\overrightarrow{AM} \cdot n = 0$$

Let $A(x_0, y_0, z_0)$ and $n = (a, b, c)$ and $M(x, y, z)$. Then

$$(x - x_0, y - y_0, z - z_0) \cdot (a, b, c) = 0$$

Equivalently,

$$\boxed{a(x - x_0) + b(y - y_0) + c(z - z_0) = 0}$$

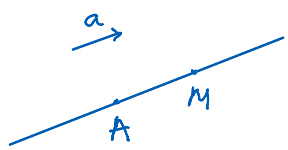
Alternatively, $ax + by + cz = \underbrace{ax_0 + by_0 + cz_0}_{d}$

Note that we could have chosen n to be of unit length. Thus, we only need to know a, b, d (3 data) to determine a plane. That is the least amount of data we need to define a plane.

Ex find the equation of a plane passing through $A(1, 2, -1)$ and perpendicular to the vector $n = (2, 1, 1)$.

Ex find the equation of a plane passing through $A(1, 2, -1)$, $B(2, 3, 3)$, $C(2, 1, 0)$.

To describe a line, we need a point on the line and a direction vector.



$A(x_0, y_0, z_0)$, $a = (a_1, a_2, a_3)$

Any point $M(x, y, z)$ on the line satisfies $\vec{AM} \parallel a$.

$\vec{AM} = t a$ where $t \in \mathbb{R}$ (scaling factor)

$$(x - x_0, y - y_0, z - z_0) = t(a_1, a_2, a_2)$$

$$\rightsquigarrow \begin{cases} x = x_0 + t a_1 \\ y = y_0 + t a_2 \\ z = z_0 + t a_3 \end{cases}$$ This is a parametric equation.

Equivalently,

$$t = \underbrace{\frac{x - x_0}{a_1} = \frac{y - y_0}{a_2} = \frac{z - z_0}{a_3}}_{\text{symmetric equations}}$$

Ex find the parametric eq and symmetric eq of the line passing through $A(1,2,3)$ and perpendicular to the plane $x - y = 5$.

Ex find the eq of the line passing through $A(1,2,3)$ and $B(2,1,1)$.

Ex find the eq of the plane containing the line $\ell: \begin{cases} x = t \\ y = -t \\ z = 3 \end{cases}$ and passing through $A(1,2,3)$.

Quadric surfaces

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fzx + Gx + Hy + Iz = J$$

Special cases:

Ellipsoid: $Ax^2 + By^2 + Cz^2 = J$ with $A, B, C > 0$

Paraboloid: $Ax^2 + By^2 + Iz = J$ with $A, B > 0$

Hyperboloid: $Ax^2 + By^2 + Iz = J$ with $A > 0, B < 0$ or $A < 0, B > 0$