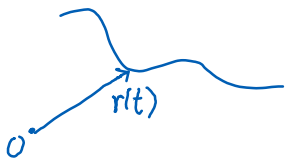
Lecture 9

Tuesday, May 14, 2024 8:33 AM

Motion problem:



$r(t)$: position vector

$v(t) = r'(t)$: velocity

$a(t) = v'(t) = r''(t)$: acceleration

$|v(t)|$: speed

$v(t)$ is always tangent to the trajectory, but $a(t)$ is generally not tangent to the trajectory

$$a = a_T T + a_N N$$

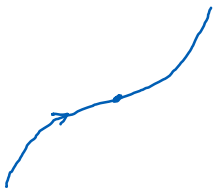
T: tangent unit vector

N: normal unit vector

$a_T = |v(t)|'$: tangential acceleration

$a_N = \kappa |v(t)|^2$: centripetal acceleration

Ex



$r(t) = (t, t^3, 0)$

$r'(t) = (1, 3t^2, 0) = v(t) \rightsquigarrow |v(t)| = \sqrt{1+9t^4} \rightsquigarrow |v(t)|' = \dfrac{36t^3}{2\sqrt{1+9t^4}} = a_T$

$r''(t) = (0, 6t, 0) = a(t)$

$r'(t) \times r''(t) = (0, 0, 6t)$

$$a_N = \frac{|r'(t) \times r''(t)|}{|r'(t)|} = \frac{|6t|}{\sqrt{1+9t^4}} = \frac{6|t|}{\sqrt{1+9t^4}}$$

At the point $(1,1,0)$,

$$a_T = \frac{36}{2\sqrt{10}} = \frac{18}{\sqrt{10}}$$

$$a_N = \frac{6}{\sqrt{10}}$$

Question: where on the curve do you get maximum centripetal acceleration?

* Functions of more than one variable:

{ domain
{ level sets (level curves, level surfaces)

* limit of functions of more than one variable:

$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = a$ if $f(x,y)$ approaches a as (x,y) approaches (x_0,y_0) in any direction (within the domain).

Ex $f(x,y) = \dfrac{xy}{x^2+y^2}$

$\lim_{(x,y)\to(1,1)} f(x,y) = f(1,1) = \frac{1}{2}$

$\lim_{(x,y)\to(0,0)} f(x,y)$ DNE

Ex $f(x,y) = \dfrac{\sin(x^2+y^2)}{3x^2+2y^2}$, $\lim_{(x,y)\to(0,0)} f(x,y)$ DNE

Ex $f(x,y) = \dfrac{\sin(x^2+y^2)}{3x^2+3y^2}$, $\lim_{(x,y)\to(0,0)} f(x,y) = \frac{1}{3}$

Ex $f(x,y) = \dfrac{xy}{x^2-y^2}$, $\lim_{(x,y)\to(0,0)} f(x,y)$ DNE