Midterm: Some problems for review

The exam will be taken at the Testing Center during 5/30 - 5/31. At the Testing Center, you go to Canvas and navigate to Midterm in Week 5 module. You will be directed to WebAssign. It will ask you for an access code. The proctor will give you the code or enter the code for you. Although all questions are automatically graded, you have an option to show your work (by typing in a box) so that you can earn some partial credit even if your final answer is incorrect.

The material covered is Section 12.1 - 12.6, 13.1 - 13.4, 14.1 - 14.7. It is a closed book exam. A scientific or graphing calculator is allowed. Phones or notes are not allowed. You will be given the following formulas at the instruction of the midterm exam:

$$\begin{aligned}
\kappa(t) &= \frac{|r'(t) \times r''(t)|}{|r'(t)|^3} \\
a_T &= \frac{d}{dt}|v(t)| = \frac{d}{dt}|r'(t)| \\
a_N &= \frac{|r'(t) \times r''(t)|}{|r'(t)|} \\
f(x,y) &\approx L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0) \\
D &= f_{xx}f_{yy} - f_{xy}^2
\end{aligned}$$

You should review the homework problems, quizzes, and examples given in the lectures. It is always a good idea to study for the exam with someone. Some problems to practice:

1) Let $a = (1, 2)$ and $b = (-3, 1)$.
   (a) Find the angle (in degree) between them.
   (b) Find the area of the parallelogram formed by them.
2) Find the center and radius of the sphere with equation $x^2 + y^2 + z^2 - 3x + 5y = \frac{1}{2}$.
3) Find the equation of the plane containing three points $A(1, 1, 1)$, $B(0, -1, 2)$, $C(3, 1, -4)$.
4) Give the correct names to the following quadratic surfaces:
   (a) $x^2 + z^2 + 4y = 0$
   (b) $-4x^2 + 25y^2 + z^2 = 100$
   (c) $x^2 + \frac{y^2}{4} + \frac{z^2}{100} = 3$
   (d) $x^2 - 2y + 2z^2 = 1$
   (e) $x^2 - 2y - 2z^2 = -1$
   (f) $-4x^2 - y^2 + z^2 = 1$
5) Find the tangent line to the curve $r(t) = (t^2 + t,\ t + 3)$ at the point $(0, 2)$.
6) Find the tangent plane to the surface $x = yz$ at the point $(2, 1, 2)$.
7) Find the tangent plane to the surface $xe^y + ye^z + ze^x = 0$ at the point $(0, 0, 0)$.
8) Find the directional derivative of $f(x, y) = \frac{x}{x+y}$ at $(1, 2)$ in the direction indicated by the angle $\theta = 30°$.