

Final Exam: Some problems for review

The exam will be on Friday June 27, 2025, from 1 PM to 3:50 PM in the regular classroom (SCB 303). It is a written exam. No books or notes are allowed. You may use a calculator (not phone or laptop) of any type. The material covered is Chapter 1, 2, 3, 5, 6, 7, 9, 10, 11, with an emphasis on Chapter 6, 7, 9, 10.

You should review the homework problems, quizzes, and examples given in the lectures. It is always a good idea to study for the exam with someone. Some problems to practice:

- 1) Write the definition of $\lim_{n \rightarrow \infty} a_n = 2$.
- 2) Write the definition of $\lim_{n \rightarrow \infty} a_n = -\infty$.
- 3) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function. Write the definition of $\lim_{x \rightarrow 1} f(x) = 5$.
- 4) Show that $\lim_{x \rightarrow 1} \frac{1}{x+1} = \frac{1}{2}$.
- 5) Prove that the sequence $\{a_n\}$ defined recursively by $a_0 = 1$ and

$$a_{n+1} = \frac{5a_n}{3 + a_n}, \quad n \geq 0$$

is strictly increasing (meaning $a_0 < a_1 < a_2 < a_3 < \dots$)

- 6) Consider the following relation on the set $A = \{n \in \mathbb{N} : n \geq 2\}$.

$$a \mathcal{R} b \iff \gcd(a, b) = 1.$$

Check if this relation is reflexive, symmetric, transitive, total, anti-symmetric, dense.

- 7) Prove that the function $f : \mathbb{R} \setminus \{1\} \rightarrow \mathbb{R} \setminus \{2\}$,

$$f(x) = \frac{2x+1}{x-1}$$

is bijective.