

## Midterm: Some problems for review

The exam will be taken in class on Friday (May 30, 2025) at the regular classroom (SCB 303) during the regular class hours (11 AM - 12:20 PM). It is a written exam. No books or notes are allowed. You may use a calculator (not phone or laptop) of any type. The material covered is Chapter 1, 2, 3, 5, 11 and Section 6.1-6.3.

You should review the homework problems, quizzes, and examples given in the lectures. It is always a good idea to study for the exam with someone. Some problems to practice:

- 1) Write the definition of a rational number.
- 2) Write the definition of the quotient and remainder in the division  $a \div b$ .
- 3) Write the definition of the notation  $a|b$ .
- 4) Write the definition of  $a \equiv b \pmod{n}$ .
- 5) Show that  $3^{2025} - 13$  is divisible by 7.
- 6) Prove that  $\sqrt{2}$  is not a rational number.
- 7) Write the definition of a perfect square.
- 8) Show that if  $a, b$  are perfect squares and  $a > 4b$  then  $a - b$  is not a prime number.
- 9) Consider the following logical statement:

$$\forall n \in \mathbb{N}, \exists x \in \mathbb{R} \text{ s.t. } x^2 > n$$

- (a) Write an equivalent English sentence.
  - (b) Is the statement true or false?
  - (c) Prove or disprove it.
- 10) Prove or disprove the statement:

$$\forall x < -\frac{1}{4}, \frac{x}{x-1} > -\frac{1}{2}$$