Math 301	
Sets: collection of items	
* use Capital letter for & Sets ,	lowerrase for elements
An item can be inside or outside of a	
	7
A set X is defined to be the set of	
all items that don't belong to itself.	Russell's
	Poradox
X={q afa} XEX? X&X	
, X∉X	
Descriptions of a Set	
· list all elements	No.
ex.	
X={1,2,4,7} order	al
= {1,1,2,4,7} repetition	
	natter
( ) DOLLIN	

list with ellipsie -	
ex.	
x= { 1,3, 5, 7,9,, 10 }	
not good:	1 / S. 7
x= 52,4,,643	30
1= 2 - 191-1-1	
Use set builder notation:	
such that	
	1 1 27
X= {natural number a a in divisib	ile by 3 g
1 st.	and the second of the
1= 51,2,3,4,5, 3 (colon)	
0 is not natural number	AF
	nckboard font)
/ - I - S n + 1 .	
Number System integers 2 - 10, =1, = 1	PIDEZ GENZ
	9 17(2) 12(1)
real numbers R -	
R= { lim an	(an) is a
η-7≈0	· Cauchy sequence,
The state of the s	(an) is a cauchy sequence, an E Q3
complex numbers C=fat	hilabe RF.
complex numbers 12-	-1
	1):2:2:2:12:16=4
quaternion  H = { ait bjtckt	d 11-j-12-j/2

Math 301

- 1) Find all divisors (positive) of 18.
- 2) Find all real numbers x such that x2-9x+20=6
  - D {1,2,3,6,9,18}
  - (2)  $X = -b \pm \sqrt{b^2 4ac}$

20

$$x = 9 \pm \sqrt{81 - 4(20)}$$

2

2

$$=$$
 9+1 9-1 10 8

X={5,4}

All positive divisors of 18 are 1,2,3,	6,9, and 18.
The lefthand side of the equation can	
(x-4) (x-5). The equation can be written as	
(x-4)(x-5) = 0. Thus, one of the two f	actors must be
zero. That is either x-4-0 or x-5=0.	211-0
Therefore, x=4 or x=5.	449 i b
hapter 2: Logic	· Jant.
	· ·
Statement is a sentence that you can ass	igh a value
of true or false.	7
> Proposition	r :
Ex: 2 is an even number True stat	enut
1 u an even number talse st	atement
1 is a better number than 2 Not	-1
iz a gener hamber than 2	P. Park
	4
7	

 $\chi = 1$ 4 is an open sentence. In other words, it would be a statement if you know the value of X. Negation of a statement Piz ~ P. (not P) Pand Q: PAQ conjunction statement Por a: PVA disjunction statement Truth Table ~ P Truth talse a PAQ F

~ (1	PAQ)	v( (~P)	ν (~a	()	77	v
P	Q	~(Pn6	()	(~P) v	(~a)	
T	T	F	1	alva?		
T	E	T			Γ	
F	1	T				
F	F	-				1
						20
Two s	itatement	are <u>logi</u>	cally equ	ivalent		, K= 3
		s have th				
value.	1					
•			W) V (~P			
P	Q	(4977)	WWW.	48CT)	esagni Edica et 👙	
T	Т		Τ			
ŧ	Т		t	600		
T	F		ŧ .			
F	F		T	N,		5. 41 - 12
1	2	3	4	5	6	3-1-1-1
P	Q	PAQ	~P	~0	(~) ~( a)	3 V 6
	T	T	F	Ŧ	F	T
7	-	F	F	T	F	F
T   T	F	1				-
T T ‡	F	<u> </u>	T	+	F	F

5. The sentence	^		
(a) "8 in ever	and 5 is pr	ime." can be wri	Hen as
(PAQ)	where P:"	8 is even" and 6	2: " 5 is prime."
TP Ta	PAQ	since both Pana	1 a are true,
1 = 1		he statement (P.	
1711		7,0,1	\
			<b>\$</b>
	Fr.		
* If P then	<u>a</u>		
P ⇒ Q			4
		•	
a multiple	4 24" can b	or multiple of 40 be written as (1) A:n is a mul	Pra)⇒R,
			lue of this sentence
depends on A	, so it is on a	open sentence, na	t statement.
		-	14
(c) If n is a	not multiphe of	10, then it is	a multiple of 2
	multiple of 5		
	(Q1(~R))?		
		_	

Mo	ath 301
Ch	apter 1
	oblem 8: Are the following sets equal?
	) I and {a: a ∈ N or - a ∈ N}, Nortural number
	The two sets, I and { a : a ∈ N or - a ∈ N}, are equal
	cause I is the set of all integers and the set { a: aeN or
~ 6	of NZ contains all the whole number, both positive and
	gative.
	) 5 1 2 2 2 2 2 2 2 3 2 4 5 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
1_(0)	) {1,2,7,3,3,3,2,2,1} oul {1,1,1,1,2,2,2,2,3,3,3,3,3}
-	Since both of the sets contain the same elements,
re	garding regardless of the how many times the elements
We	re repeated, the sets are still equal.
(c)	of did is a day with 40 hours } and fw: wie
	a week with le days }
*	f you have a long set, give it a name.
_£X	
	et A= { a! a EN or & a- a EN }



conditional statement:	
0 7 0 " P cinalizes &	L"
typhoteris cordusion " D whenever	P "
interedant consequence " If P then I	a"
"P only if	
If n is a prime number	then no is a prime number.
·Þ	Q.
-	
P=7 Q is false only when 1	is true and a is false.
PQP=7Q	If 1=2 then 2=3: P=>A
TTTT	P; "1=2"
TFF	Q:"2=3"
FTGT	There fore it is true statement.
FFBT	1100 Joseph 11 June 12 June 12 11 June 12 11 June 12 June 12 11 June 12 June 12 June 12
-	

conditional statement p=7 Q.

QPP: converse of the above statement.

(NP) => (~Q): inverse of the above statement

(-Q) = 7 (~P): Contrapositive of the above statement

(P=7Q) and ((VQ)=7(~P)) are logically equivalent.

(QFP) and (NP=7~03 are logically equivalent.

P <= 7 Q means (P=>Q) ~ (Q=P)

Pin equivalent to D.

P it and only if a.

P iff Q.

P is a necessary and sufficient condition for Q.

P	Q	P>a	Q⇒P	POR	
T	T	T	T	T	
T	F	F	T	F	
F	.7	T	+	F	
P	+	T	T	T	

Chapter 2 problem 6 (a)  $(x \in R) = 7 (x^2 \in R) \wedge (x^2 > 0)$  $\neq$  Let  $P: (X \in R)$  and  $Q: (X^2 \in R) \land (X^2 > 0)$ . 1 x let P = (XER) and Q = (x2ER) 1 (x2>0) If x is an element of the set of all real numbers then x2 is an element of the of all real numbers and x2 is greater than 0. If x is a real number then x is a real number and x2 is greater than 0. (b) 4 E { 2l : l EN} 4 is a positive even number.  $(c) (x \in N) \Rightarrow \sim (x^2 = 0)$ If x is a natural number then x2 is not equal to o. (d) (XEI) => (XE {2l: lEI}) V (XE [2k+1]: KEZ)

Math 301	
* Practice wri	ting definitions
Even	
- An integer	n is even if $n=2k$ for some $k \in \mathbb{Z}$ .
In a definition	n, "if" is the same as "if and only if"
Ex' Oo Chitha	for parent cause.
n is a perf	Fect square if $\sqrt{n}=9$ K for some KEI.
	a perfect square if A=)
=xina	
_ for definition	m,
	nare root (V) is not too friendly.
z,y,X -> for	real numbers
	natural, integers
p -> print	
* A number	- $n$ is a perfect square if $n = a^2$ , for
some a EI	2.
	-2/3
	3/2/0
	·.\

Exercise.	
Write the definition of	
i) a prime number	
2) a u divisible by b	
3) quotient and remainder of a division a + b	
where $a \in \mathbb{Z}$ , $b \in \mathbb{N}$ .	
(1) A number p is a prime number if	
- p has only a factors, I and liftely	
p	
(2) a indivisible by A number a is divisible by	6
such that a EZ if a for some bEN.	
south that if at I.	
(3)	-
Answer key:	-00
(1) A prime number is a natural number greater than	1 which
has only two divisors: I and itself.	
J	

(r<6)ew
(3) A number 9 & I is a quotient and rEN is a
remainder of the division $\frac{a}{b}$ if $q = q(b) + r$ .
r FN, where r <b,< td=""></b,<>
* The islue of this definition is that
b hasn't mentioned yet.
A number q EI is a quotient and r EI is a
remainder of a if $a = q(b) + r$ and $0 \le r \le b$ .
Most statements you are asked to prove
are of the form "P= a"
Trivial cases: Pir false or Q is true.
In either cases, the implication P => Q is true.
Ex: Prove that if $x^2+1=0$ for some number $x \in \mathbb{R}$ then
$\chi^2 = 0$ .
a P
N

pirect proof	
· you assume the hypothesis P is true.	
. Then you use a chain of logic to arrive at Q being true.	S
x: Prove that if nEZ then n2+2n+1 is a perfect square.	
Assume n E I	9
we can factor n2+2n+1 as follows.	
$n^2 + 2n + 1 = (n+1)(n+1) = (n+1)^2$	
Since NET, then not e I as well.	
Therefore $(n+1)^2$ is an integer, which by definition is	
a perfect square.	
	-
	-15
w. ·	

Math 301 Prove += a by direct proof. · Assume P is true. · Use a chair of logics to imply a a true. Write " a = b (mod n)" (a is congruent to b if a-ban divisible by n. in modulo n) 46-a 4=1 ( mod 3 ) Ex: (mod 2) 5 = 17 27 51 PSX with 913x Observation: a = 6 (mod n) if and only if a and b have the same remainder in the division by n. Ex: Prove that if n=2 (mod 5) then n2 = 4 (mod 5) · Assume  $n \equiv 2 \pmod{5}$  is true. We want to show that  $n^2 \equiv 4 \pmod{5}$ We have n-2 is divisible by 5. There is kEI such that Alternatively, we can say "So, 11-2 = 5k for some k & Z."] We want to show that n2-4 is divisible by 5. (That is to show n2-4 = 5k for some to EZ.) ( This is wrong since k has already been used! That is to show n=4=5e for some e E Z. We have n = 2 + 5k. Equaring both sides gives  $n^2 = (2+5k)^2 = 4+20k+26k^2$ . So, n2-4 = 20 k + 25 k2 - 5 (4 k + 5 k2) We can choose / We have found 1 = 4k + 5k2

	158	H36-941
Fact	: for any $x \in \mathbb{R}$ , we have $x^2 \ge 0$ .	
	puritant of BE	ywil-
Fact:	If a ≥ 0 and brand b>0 then atb>0.	
	and 5 4 Ju	2/11
Fact:	If a > b and c> 0 then act bo! A then it sught to reals a	,_1) ·
	If a>b and CLO then accbc.	
	of al provinces is is (a from) of the "	slicitl.
Fact:	If x>0 then 1 50 dayson is in us adjusted it got-1	11
	If x<0 then 1<0.	
Fact:	If a > b then atc > b+c.	5/4
	(= han) FI = 3	
	Ex: Prove that XER then X2+1>2x.	
	Assume that XER is true. We want to show that x2+1 \ 2x.	nd5
4	(We have for any XCR, we have X2 >0.) I have to all the second	
	(We note for why we by the state of the stat	
	We have the many very week and a contract of the contract of t	
	We have for any XER, He have x 2000	
	So) we can write x2+1 > 2x as follows:	î zaî
	So we can write $x^2+1 \ge 2x$ as follows: $x^2+1-2x \ge 0$ ; from $y = 0$ with $(x^2+1) \ge 0$ and $(x^2+1) \ge 0$	î 33
	So we can write $x^2+1 \ge 2x$ as follows: $x^2+1-2x \ge 0$ ; from $y = 0$ and $(3^2 + 1.0)$ and $(3^2 + 1$	
	So we can write $x^2+1 \ge 2x$ as follows: $x^2+1-2x \ge 0^2$ , from $y = 0$ and $(3^2 + 1.0)$ and $(3^2 + 1.0)$ and $(3^2 + 1.0)$ and $(3^2 + 1.0)$ This is equivalent to $(3^2 + 1.0)^2 \ge 0$ and solve at depart of solve it (3 by a) $x = 1$ and $x = 1$	2 H J
	So we can write $x^2+1 \ge 2x$ as follows: $x^2+1-2x \ge 0$ , from $y = 0$ and $(3^2 + 1)$ and $(3^2 + 1)$ and $(3^2 + 1)$ This is equivalent to $(3^2+1)^2 \ge 0$ and so the solution of specific $(3^2 + 1)^2 \ge 0$ and $(3^2+1)^2 \ge $	en o Lak
	So we can write $x^2+1 \ge 2x$ as follows: $x^2+1-2x \ge 0$ , from $y=1$ [Figure 1.3] This is equivalent to $(x=1)^2 \ge 0$ . It solves to describe the form of secret is (2 logar) and $(x=1)^2 \ge 0$ . There fore, it is greater than $x=1$ .	₹# 1 
	So we can write $x^2+1 \ge 2x$ as follows: $x^2+1-2x \ge 0$ , from $y=1$ [Forewir] This is equivalent to $(x=1)^2 \ge 0$ . It solves to describe the solves of solves to the solves of solves of solves of the solves of	2 H J L - M - M - M
(e 2)(n	So we can write $x^2+1 \ge 2x$ as follows: $x^2+1-2x \ge 0$ , from $y=1$ [Forevis] This is equivalent to $(x=1)^2 \ge 0$ . It solves to describe the solves of solves to the solves of solves to the solves of sol	2 A J
(e 2)(n	So we can write $x^2+1 \ge 2x$ as follows: $x^2+1-2x \ge 0$ [Forevii) This is equivalent to $(x=1)^2 \ge 0$ Since $x \in \mathbb{R}$ I, then $(x-1)^2$ is non negative. In the second of the seco	2 2 X
fe over	So we can write $x^2+1 > 2x$ as follows: $x^2+1-2x > 0$ This is equivalent to $(x=1)^2 > 0$ Since $x \in \mathbb{R}$ I, then $(x-1)^2$ is non negative. Therefore, if is greater than zero is the property of the	2× 2× 2× 2×7-15
/e 2/m	So we can write $x^2+1 \ge 2x$ as follows: $x^2+1-2x \ge 0$ Figure 1. This is a equivalent to $(x^2+1)^2 \ge 0$ Since $x \in \mathbb{R}$ Therefore, if is greater than zero 1. In the property of the	2× 2× 2× 2×7-15 2-15
(e 2)(n	So we can write $x^2+1 \ge 2x$ as follows: $x^2+1-2x \ge 0$ This is equivalent to $(x\pm 1)^2 \ge 0$ Since $x \in \mathbb{R}$ Therefore, it is greater than $2 = 0$ $2 = 0$ Therefore, it is greater than $2 = 0$ $3 = 0$	2× 2× 2× 2×7-15 2×7-15 3
(e 200)	We can write $x^2+1 \ge 2x$ as follows: $x^2+1-2x \ge 0$ ; home $y=0$ and $y=0$ . This is equivalent to $(x^2+1)^2 \ge 0$ . It solves the short of south is $(x^2+1)^2 \ge 0$ . Therefore, then $(x-1)^2 = 0$ and non negative. List is $(x^2+1)^2 \ge 0$ . Therefore, it is greater than zero of the solves of production $(x^2+1)^2 \ge 0$ . Therefore, it is greater than zero of the solves of production $(x^2+1)^2 \ge 0$ . Therefore, it is greater than zero of the solves of $(x^2+1)^2 \ge 0$ . Therefore, it is greater than zero of the solves of $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is	2x 2x 2x 2x 2x7-15 2-15 3 2-5
Te 200	So we can write $x^2+1 \ge 2x$ as follows: $x^2+1-2x \ge 0$ This is equivalent to $(x\pm 1)^2 \ge 0$ Since $x \in \mathbb{R}$ Therefore, it is greater than $2 = 0$ $2 = 0$ Therefore, it is greater than $2 = 0$ $3 = 0$	2x 2x 2x 2x 2x7-15 2-15 3 2-5
fa: 2 on	We can write $x^2+1 \ge 2x$ as follows: $x^2+1-2x \ge 0$ ; home $y=0$ and $y=0$ . This is equivalent to $(x^2+1)^2 \ge 0$ . It solves the short of south is $(x^2+1)^2 \ge 0$ . Therefore, then $(x-1)^2 = 0$ and non negative. List is $(x^2+1)^2 \ge 0$ . Therefore, it is greater than zero of the solves of production $(x^2+1)^2 \ge 0$ . Therefore, it is greater than zero of the solves of production $(x^2+1)^2 \ge 0$ . Therefore, it is greater than zero of the solves of $(x^2+1)^2 \ge 0$ . Therefore, it is greater than zero of the solves of $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is	2x 2x 2x 2x 2x7-15 2-15 3 2-5
(x: 2'0n	We can write $x^2+1 \ge 2x$ as follows: $x^2+1-2x \ge 0$ ; home $y=0$ and $y=0$ . This is equivalent to $(x^2+1)^2 \ge 0$ . It solves the short of south is $(x^2+1)^2 \ge 0$ . Therefore, then $(x-1)^2 = 0$ and non negative. List is $(x^2+1)^2 \ge 0$ . Therefore, it is greater than zero of the solves of production $(x^2+1)^2 \ge 0$ . Therefore, it is greater than zero of the solves of production $(x^2+1)^2 \ge 0$ . Therefore, it is greater than zero of the solves of $(x^2+1)^2 \ge 0$ . Therefore, it is greater than zero of the solves of $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is $(x^2+1)^2 \ge 0$ . The solves of $(x^2+1)^2 \ge 0$ is	2x 2x 2x 2x 2x7-15 2-15 3 2-5

	5/12/2025	
	Ex: Show that if $ X  \leq 5$ then $\frac{x+3}{x} > \frac{2}{5}$	tingsitted of part
	Х Б	
		) <- , () <- 1).
	Assume that IXI < 5. We want to show that	x+3 > 2
in the same	Print A 19	x <u>5</u>
	we can write X+3 2 - Collows:	5
	We can write $\frac{x+3}{x}$ $\frac{2}{5}$ as follows:	Hint: $\frac{a}{1} > \frac{c}{1}$ (*) Method
		1 11 1 b madely
	$\frac{x+3}{x} - \frac{2}{5} > 0$ .	If bod > 0 then (*) is
		equivalent to ad > bc.
	Contract of the bound	equivam 1. 40 > 86.
	$50,  5(x+3)-2x \rightarrow 0$	10.44 63.5
	5x	If bd <0 then (x) u
	Then we fingstify, compute the	equivallet to ad < bc.
	numerator	15
	(1) Flui !=	Method 2
	5x+15-2x 70	(*) is equivalent to
	5x	
	3x + 15 > 0	$\frac{a}{b} - \frac{c}{d} = 0$
	5x	equivalent to ad-60 > 0
	Then, we simplify:	Then show that ad-be and
	3x +15 >0	bd have the same sign.
	15 4-3x _ 5 7-x	
-	3 3	
	Since 1x1<5, then 5>1-x1 is 5>	· lx l
	Since XXX, The STITLE ST	TNI.
		The state of the s
		i i i i i i i i i i i i i i i i i i i
0		

	5/14/2025	
	Proof by Contrapositive.	
	3 X	
	(P=7a) <=> (~a ⇒ ~P)	
	= , a fit is least man of tracted out to the project.	
	Ex: show that if nEN then n2+4 is not a perfect square.	
\$	The state of the s	
A HAR.	B a day	
	Assume nº+4 h a perfect square.	
211	with the least of	
. 5 1 × 1.5	Ex: Show that if 3 Yn then n2 = 1 (mod 3).	
	9-7-1:-(11/18) 02	
ji (x)	Direct proof.	
	Then is completely compact the second of the second of	
	Assume 3 km. We have two cases.	
	Case of sit n = 1 (mod 3)	No.
\$\frac{1}{4} -	1 -1 = 3k for some KEZ	
	0 = 3k+/:11 = 0 = 1 = 1 = 1 = 1 = 1 = 1 = 1 = 1 =	
	There is a remainder of 1, 3 to x5	
T Y J0 -	to of the bridges XZ	
100	Case 2: $n \equiv 2 \pmod{3}$	
La same	tool must will = 11-2= 36 for some let.	
	n=31+2	
	There is a remainder of 2 X8-201	
	n = 3kf/2 is  x-152 - 1 n=3kt2.	
	$(31+1)^2 - 1 = 3$ (31+2)^2 - 1 = 31	
	$9l^{2}+6l+1-1=31$ $9l^{2}+12l+4-1=3l$	
	6121Ch = 3' 12-+121+3=31	
•	$3k(3k+2)=31$ . $3(3e^2+4l+1)=3l$	
	71 4 6 2 1	
	$3k = -1$ $l = 3l^2 + 4l + 1$	

Assume 3 /n. We have two cases. case 1: n=1 (mod 3). This can be written as n-1 = 3k for some k \( \mathbb{Z} \). 20, n= 3k+1. There is a remainder of 1. Case 2: n = 2 (mod 3). This can be written as n-2=3l for some LEZ. So, n: 32+2. There is a remainder of 2. contruction We want to show  $n^2-1=3m$  for some  $m \in \mathbb{Z}$ . We have n= 3k+1. 50,  $(3k+1)^2 - 1 = 3m$ , appoint a which This is equivalent to 9k2+ 6k = 3m. To further simplify (a) | H ] (a bound d = b  $m = 3k^2 + 2k$ We have found com = 13 k2 + 12 k years of instances north (a bown) b = > box (a boxa) d= p 1] We want to show n2-1-37 for some r & I. We have N = 30+2 (4 bush) bd = 15.  $50, (3l+2)^2 = 3n.$ This is equivalent to 912+121+3= 3r. To fin ther symplify.

r = 3l 2+4l+1. We have found n = 322+48+1

a=b (mod n) (-7 a2=62 (mod n) 29 = 26 (mod n)  $\binom{a \equiv b \pmod{n}}{2 \equiv 2 \pmod{n}}$ Ex: What is the remainder of 3 2015 in the division by 20? (Mod 20) (34)506 = 1506 = 1 (mod 20) (mod 20) 3 ( mod 20)

## Absolute value

$$|x| = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases}$$

## Properties:

• 
$$\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$$
 for any  $x, y \in \mathbb{R}$ ,  $y \neq 0$ .

· Ix" = Ix1" for any xER, nEN

and nEM. · IXI > x and IXI > -x for

1x+u1 & 1x1+1y) for any x, y & IR.

## Ex: Let x, y & R. Prove that

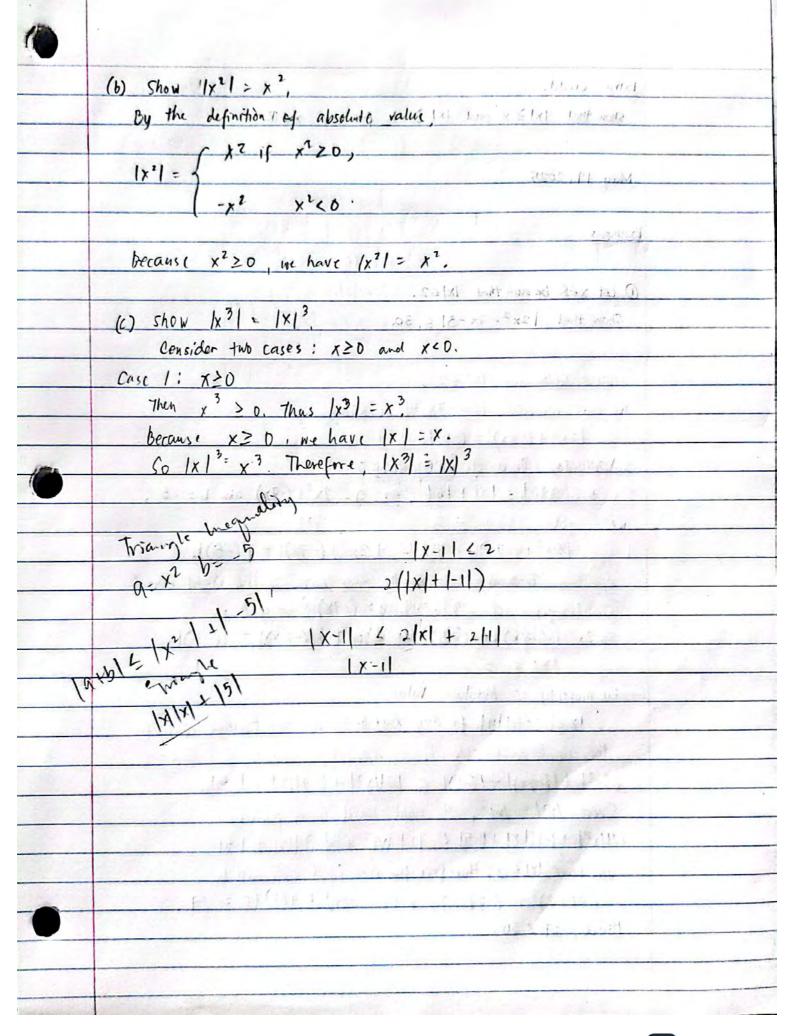
(a) 
$$|-x| = |x|$$
.

(b) 
$$|x^2| = x^2$$
  
(c)  $|x^5| = |x|^3$ 

Assume that x, y f IR. Show that 1-x1=x, 1x21=x2, 1x31=|x31, and 1x-41 2 1x1-141.

$$= |x|$$





Extra Credit:

show that IXI > x and IXI > -x for any xER.

May 19, 2025

Examples:

D Let XER be such that 1x1 = 2.

Show that 12x2-7x-51 4 30.

Assume XFR and IXI & 2.

we can rewrite 12x2-7x-51 as follows:

 $|2x^2+(-7x)+(-5)|$ 

Applying Triangle Inequality:

| a+6| \( |a| + |b| , for a = 2x2+ (-7x) and b = -5,

we get:

 $|2x^2+(-7x)+(-5)| \leq |2x^2+(-7x)|+|(-5)|$ .

Applying Triangle Inequality once more to the right hand

side for a = 2x2 and b = (-7x), no get

 $|2x^{2}+(-7x)|+|(-5)| \leq |2x^{2}|+|(-7x)|+|(-5)|$ .

By property of Absolute Value,

1xy1 = |x||y| for any x,y ER, we can further simplify

the right hand side to

 $|2x^2| + |(-7x)| + |(-5)| \le |2||x^2| + |-7||x| + |-5|$ .

Since 1x21 = 1x12, the right hard is my follows:

1211/21 + 1-711 ×1 + 1-51 1 12 1412 + 1-7 141 + 1-51

we have 1414 2, therefore the right had side will be:

121(2)2+ 1-71(2)+1-51=2(4)+7/2)+5=27.

Hence, 27 / 30.

2 Let XETR be such that 1x-11 & 2. Show that 1x2-51 < 15. Tony The hypotheris is P: (x(R) A (1x-1142). The conclusion is Q (1x2-51 <15) We need a to show P=> Q. Assume XER and 1x-1162. Show that 1x2 51 < 15. We have 1x-1/22. Add 1 to both sides: 1X-11+1 62+1. Ne non have :  $|x-1|+1 \leq 3$  (1) Applying Triangle inequality (atb) = (al+1b) (\*) for a = x-1 and b=1, we get :  $|(y-1)+1| \leq |y-1|+1$  (2) By (1) and (2) simplifying LHS(2), we thate: We want to show 1x2-5/215. Applying Triangle inequality (\*) for a = x2 and b = -5. we get to 1 1 1-5| by product Rule, 1x21 = |x||x| for any Xt/R. m2 +1 = 5k



May 20, 2025

Proof by contrapositive:

 $(P \Rightarrow Q)$  is equivalent to  $(\sim Q \Rightarrow \sim P)$ direct contrapositive

Proof by contradiction:

(P=7 a) is equiv. to (PA-a) = contradiction

a is true is equivalent to (NA => contradiction).

Ex: Prove that  $\sqrt{6}$  is irrational.

Suppose by contradiction that VG is national. Assume

So,  $\sqrt{6} = \frac{9}{6}$  for some  $a, b \in \mathbb{Z}$ ,  $b \neq 0$ . We can assume that the bracking  $\frac{9}{6}$  is in simplified form. (We can assume that

GCD (a,6)=1).

By squaring both side, we have get ottain:

 $6 = \frac{q^2}{L^2}.$ 

multiply both sides by b2, we get

 $(b^2 = a^2. (1)$ 

Because a2= 2(362) in an even number,

a must be an even number. So, a=2c

for some c EZ.

Substituting a=2c into (1),

we get  $6b^{7}$ .  $(2c)^{2} = 4c^{4}$ ,

which is equivalent to

 $3b^2 - 2c^2$ .

So, (Becomse) 362 is an even number.

So, bi in an over number.

So, b is an oven number.

So, b= 2d, for sine dETL.

**Another example:** show that  $\sqrt{2} + \sqrt{3}$  is irrational.

Suppose by contradiction that  $r = \sqrt{2} + \sqrt{3}$  is rational. Then  $r^2 = 5 + 2\sqrt{6}$  is also rational. Then  $\sqrt{6} = \frac{r^2 - 5}{2}$  is also rational. This contradicts the fact that  $\sqrt{6}$  is irrational.



Quantified Statements	
	turing to he
Statements that start with	
"There exists " or " for all".	are many and it is found
Ex: There exists nfN such that	part e and (a)
n2=6 (mod 10). (True)	201, 15 T
( u f	with the state of
Ex: For all nEN, 60 2"-1 is p	orime. (talse)
	n tale that : That he have
Notations:	in the state of th
7 n € N s.t. n2 = 6 (mod 10).	Log it not
V ¥ n∈N, 2 <sup>n</sup> -1 is prime.	
	Vac A The First lead force i
(3!)n EN s.t. n2=6 (mod 10)	Henry to prove
There exist only one	
	Frankylast :
] n,k ∈ N s.t. n2 = 6 (mod 10) and	k2 = 6 (mod 10)
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STATEMENT	
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We now to find ne A such that	P(n) is true.
Yn EA, P(n): For all leach levery.	MEN, I(I) WINE.
How to prave?	(3) REM S.t. Mish (mid 10) - 11
	67 A - 1
Pick any nEA. We will show that	P(n) is true and the first south
Sautisla d'	9470
Santial of t	P(n) is true
For each nEA, we will show that P(n	The frue (al Lou) DE in . 12 Made E
For each nEA, we will show that P(n 7(a): We need to find XEZ such to	a+10
quivalent: For each $nEA$ , we will show that $P(n = 7(a))$ : We need to find $xEZ$ such to $xEZ$	hat there exists 19 + Z such that In E
For each $nEA$ , we will show that $P(n = 7(a))$ : We need to find $xEZ$ such to $x + y = 3$ .  Pick $x = 0$ . We need to find $xy$	hat there exists 19 + Z such that In E
tor each nEA, we will show that P(n  7(a): We need to find xEI such to  x+y=3.  Pick x=0. We need to find my  We can choose y=3. ■	hat there exists 19 EZ such that
For each nEA, we will show that P(n 7(a): We need to find XEI such to X+y=3.  Pick X=0. We need to find ary We can choose y=3.	hat there exists 1962, such that in E  61 Such that o'try=13.
tor each nEA, we will show that P(n  7(a): We need to find xEI such to  x+y=3.  Pick x=0. We need to find my  We can choose y=3.  ■	hat there exists 1962, such that the E
tor each nEA, we will show that P(n 7(a): We need to find xEZ such to x + y = 3.  Pick x=0. We need to find ny We can choose y=3.  A(c.) for all xEZ, then exists y	hat there exists 1962 such that the E  62 Such that 1949=13.
For each nEA, we will show that P(n 7(a): We need to find xEI such to x+y=3.  Pick x=0. We need to find right we can choose y=3.  Pick any xEI, then exists y	hat there exists \( y \in \mathbb{Z} \) such that \( \text{The exists} \) \( \text{Y} \in \mathbb{Z} \) such that \( \text{The exists} \) \( \text{Y} \) \( \text{Z} \) such that \( \text{The exists} \) \( \text{The exists}
For each nEA, we will show that P(n 7(a): We need to find xEI such to x+y=3.  Pick x=0. We need to find ry We can choose y=3.  P(c.) For all xEI, then exists y  Pick any xEI. We will show that	hat there exists 1962 such that the E  62 Such that 1949=13.

BXEA S.t. P(x). Negotion: IR was to me Grant of Charle YX tA , ~P(x). + xEA. P(x). ( ILE / Negation: ( ) 15 / 12 / 12 / 12 / 12 3xEASL~P(x). 1/2 - = = (2 = 1/2). We now to the 2/2 = = = = = 1/2 TXEA. ( FYED s.t. 1(x,y). Negation: FREA S.L. Y YEB, ~ 1(x, y). Mile ne 2. M read to show that a TOO 112 ( end 2) (thou) of a year observed (show) 1 = 1 = 1 a with Oliver 1 Ton 18 290

CAN IN ADXE May 28, 2025 No grant. Vail I if 3 (a2+62) then 3 a and 316. Pick a, b & I. We want to show (that if 3 (a2+62) then 3/a al 3/6) C+ can be replaced by 3/(a2+62)=X(3/a) 1 (3/6). Assume 3 (a2+b2). We need to show 3/a and 3/b. the following Kemma: 12. HELD ) , ASOF To do this, we will first prove that ... ] lemma Yn € Z , n2 = 0 or 1 (mod 3). (0) 1 or 434 Y . d. x A3 15 Pick n \ 2. We need to show that n2 = 0 or n2 = 1 (mod )). We consider 3 cases n=k (mod 3) for k=0,1,2. Case 1: n = 0 (mod 3) Then n= 02 = 0 (mod 3). Case 2: n=1 (mod 3) Then n2 = 12= ( (mod3). Case 3: n=2 (mod 3) Then n2 = 4 = 1 (md 3). Therefore, the Lemma has been proven. Let us go back to the problem. By the lemma, a== 0 m 1 (mod 3). 62 = 0 or 1 (mod 3). So, a2+b2=0 only in the case a2=0 and b2=0 (mod 3). So, and 3/a2 me 3/62. So, 3/a and 3/6.

-4	a.) Consider the following logical statement: True
	YNEN, BRER s.t. x2>n For all n is a Natural number, there exists a real number x such that x2 is greater than n.
	Pick any $n \in \mathbb{N}$ . We will show that there exists $x \in \mathbb{R}$ s.t. $x^2 > n$ .
	We need to find $x \in \mathbb{R}$ such that $x^2 > n$ .
	Let use 5- n+1 Ve hours:
-	sincer the, we can choose not the contract of the things
	Since 72 > R. for choose x= n+1. Vehove: (n+1) <sup>2</sup> >n ~ (n+1) <sup>n</sup> = x+2n+1 = xh + n+n+1. y <sup>2</sup> +2n+1>n DeOtato 24
-	
	Let us choose x=n+1. We have and the MF OF P
_	$\gamma^2 = (n+1)^2 = n^2 + 2n+1 = n^2 + n+1 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 +$
	(4) 1 (4) 1
	()(W)() = 1
_	के दि कि कि के कि कि कि
_	1
	DO OF F F F F F F F F F F F F F F
_	10. Prove or disprove the statement:
	$\frac{1}{4} \frac{\sqrt{x} \sqrt{x} - \frac{1}{4} \frac{1}{\sqrt{x} - 1}}{\sqrt{x} - 1} > \frac{1}{2}$
	the sequence (m) something the sequence (m)
11	The design september landering to the first and the first
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	at seen gardinongs

1 2 3 4 5 6 7 10 N. M. ... & a last want inter that is Ex: Let an = 2n+1 Prove that  $\lim_{n \to \infty} q_n = \frac{2}{3}$ Pick any  $\mathcal{E} \in (0, \infty)$ . We want to find  $N \in \mathbb{N}$  s.t. Assume  $n \in \mathbb{N}$  and  $n > \mathbb{N}$ . We have  $\frac{2n+1}{3n+2} = \frac{3(2n+1)-2(3n+2)}{3(3n+2)} = -1$  3(3n+2)Taking the absolute value of both sides, we get  $\frac{|2n+1|-2|-|-1|}{3n+2-3} = \frac{|-1|-3|}{|3(3n+2)|} = \frac{|(474)(5-3)|}{|3(3n+2)|} = \frac{|474|}{|3(3n+2)|} = \frac{|474|}{|3(3n+2)|} = \frac{|474|}{|3(3n+2)|} = \frac{|474|}{|3($ ause  $n>N_3$   $\frac{|2n+1|}{|3n+2|} = \frac{1}{3} \left(\frac{|3n+2|}{|3n+2|} + \frac{|4|(2-3)|}{|3|}\right) = \frac{1}{3} \left(\frac{|4|}{|4|} + \frac{|4|}{|4|}\right) = \frac{1}{3} \left(\frac{|4|}{|4|}\right) = \frac{1}{3} \left(\frac{|4|}{|$ Because n>N, (P/8) / > To ensure that  $\left|\frac{2N+1}{3N+2}\right| \leq \epsilon$ , we need to pick NEN such that  $\frac{1}{3(3N+2)} \leq \epsilon$ . This inequality is equivalent to 3(3N+2) > 1 Which is equivalent to 3N+2 > 1 , Which is equivalent to N 7 & ( 1/3 E - 2 ).

Floor function:
N W s e s e s s e s s e
[a] u the largest integer that is
smaller than or equal to 9.
[2.7] = 2 [5] = 5
[102] = 10 [-5] = -6
to Mark the st form sit, (co. s) a y y y to some
We pick $N = \left[ \frac{1}{3} \left( \frac{1}{3\epsilon} - 2 \right) \right] + 1$
* We pick $N = \max \left\{ \left[ \frac{1}{3} \left( \frac{1}{3\xi} - 2 \right) \right] + \left[ \frac{1}{3\xi} \right] \right\}$ . As a more complete.
(4+n=)0 (5+n=) 3 (3n+z) = 0(+n=)
The second section of the section of t
Total the alcologic volume of Light sides, percent
$ x^2-4  =  (x-2)(x+2) $
$ x^{2}-4  =  (x-2)(x+2) $ $=  (x-2)(x+2) $
€ 8 1×+21
$= \frac{1}{4} \left\{ \frac{(x-2)+4}{4} \right\}$
≤ 8 (1x-21+141) (21x2) € (41x8) € (4x8)
< <u>\$</u> (\$+4)
LANK LOVE MICH AND ST LOW LOW TO THE STATE OF THE MICH MICH.
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June 4, 2025 (IN + 15-41) & -
(++1) (x)
Definition!
Let f be a function of real variable.
we say that f converges to L as x tends to a if:
VETO, 38>0 st. VXER, a-81x1 918 => L-81f(x) & L+8.
* For any positive E, we can find a
positive number & Isnch that as long
as x is within & away from a,
f(x) will be within & away
from L.
$x = \frac{3}{2} 7 \times 27 = (4+1) \times (4+1) \times 2$
We (need) to) write like like this:
YE70, ∃ 870 s.t. (9-81 X < a+8) A (x≠a) => L-E < f(x) < L+E.
 0 <  x-a  < 8  f(x)-L  < E
Ex: Prove that the $\lim_{x\to 7} x^2 = 4$ .
Answer:
Ψετο, 3870 s.t. 0<  x-2 < 8 =>  x <sup>2</sup> -4 <ε.
Pick any \$70. We want to find \$70 such that $0 <  x-2  < \delta = 7  x^2-4  < \epsilon$ .
$0 <  X-2  < \delta = 7  X^2-4  < \epsilon.$ hew line
Assume 06/x-2/6 [we want to short 1x2 4/ 5] EFROR
we have,
$ x^2-4  =  (x-2)(x+2) $
=  (x-2)  (x+2)
4 8   X + 2
= \( \( (x-2) \) \ \ \ \

48 (1x-21+141) < 8 (5+4). We want to find 8>0 such that sharpy har to notioning p of 7 fol We can that f consequer to I or when to be if : 3> (++ 2) 2 Choose  $\delta = \frac{1}{2} \min \left\{ 1, \frac{\epsilon}{3} \right\}$ Then  $S \leq \frac{1}{2} \leq 1$  and  $S \leq \frac{1}{2} \leq \frac{\varepsilon}{2}$ . We have  $\xi(\xi+4) < \xi(1+4) = \xi < \xi = \xi$ 3 2 14-(x)=1 8 2 16-x120 PICK CAN SEC. HE MANTED FOR SEC SUL MOST DC | K-2 | CB =2 | K2-1 | CE. Axin caralle of roma sign 3 x15-x120 mucch 16=+x)(= x) = |+== x1 1124011(=-1)1 = 124213 2 1++(1-1)/2 =

June 6, 2025 Example of proving the limit of a sequence doesn't exist.  $a_n = (-1)^n$ show that lim an doesn't exist. Method 1: Supposed the limit exists. lby contradiction that Let Riomand L = lim an. We have YE>O, FLORD = NEW St. MEN, N>N ⇒ |an-L| CE. We have  $|a_n-L|=|(-1)^n-L|=$   $\begin{cases} |I-L| & \text{if } n \text{ is wen} \end{cases}$   $\begin{cases} |I-L| & \text{if } n \text{ is odd} \end{cases}$ scratchnork 11-41 1-1-11 Pick E = 1 . There exists NEN such that Yn∈N (and), N>N => |(-1)"-4/<3. For n= 2N, we have |(-1) = - | < \frac{1}{3}. In other words, 11-L1<\frac{1}{3}. O for 1= 2N+1 ..... 1-13-L/< 3. 10 -11+ -11+

(1) implies - 1 < 1-L < 1, which implies 2 < L < 4. 1 implies - 1 <-1-1 < 1 , which implies - 4 < L < -2 We have This is a contradiction. trovo dono 112 12-11 Mathematical Induction P(n) is a statement depending on n. If · P(a) is true, · YKEZ and king, P(k) => P(kH) is true, then P(k) in true for all kEZ, k > q. tx: Show that fir all n 25, 2" > n". P(n): "2" > n" P(5): "32725". P(5) is true t the base case 1 werrant to snow Yk>5, P(+) -> P(++0. Prot KEZ, \$25. Want to sur P(4) => P(41). Assume P(k) is true, we want show P(K+1) also true.

have: 21 > k2 Show: 2kH > (k+1)  $(k+1)^2 = k^2 + 7k+1$ June 9, 2025 Ex: Show that 2" 7 n2 YnEN, n26. I can be interchanged We show by induction on n = 5 that  $2^{n} 7 n^{2}$ . (1) for n=5, we have  $2^5 = 32 > 25 = 5^2$ . so, (1) is true for n=5. suppose that (1) is true for n= & for som K ≥ 5. We have 2k > k2. (2) We mand to show that 2 k+1 > (k+1) (3). We have / Because of (2), we have 2k+1 = 2.2k 7 2k2 To prove (3), it is sufficient to show that  $3k^2 \ge (k+1)^2$ . (4) (4) is equivalent to  $k^2 \ge 2k+1$  (5) k2.2k - k(k-2) > 5.3: 15 >1. Therefore, (5) is true.

26. Let 90, a, 92... be a sequence recursively defined as do= 2, 9,=1, and  $a_n = a_{n-1} + 6a_{n-2}$  for  $n \ge 2$ . Prove by induction that  $a_n = (-2)^n + 3^n$  for an  $n \ge 0$ We show by induction that m N20 that  $a_{N} = (-2)^{n} + 3^{n}$ Ex: Show shad 2' > n & Vn & N & N & " can be interchanged We stud by induction on all that 2" 7 " 2 = 32 > 25 = 52 so, (1) is true for no s. suppose that II) is this for not for you was the use egal to shim that (a) (141) ( (a). We have I Felows of (2), we have To prove (3), it is sufficient to show that I well short 150 : 100 \$ (8-1) 4 - 16.51 Thurston (A) is spent

Ex: Let a, az, az , ... be a sequence defined recursively as a=3 and for all n=2, 99 = 2an-1-1. Prove that an = 2"+1 for all nEN. we show by induction that on n 1 1 that  $a_n = 2^n + 1.$  (1) for n= 1, we have graphins. gaztraz.  $q_1 = 3 = 2^l + 1$ . So, (1) is true for n=1. Suppose that (1) is true for n=k for some k≥1. we have  $q_k = 2^k + 1$ . (1) We want to show that ak+1 = 2 +1 +1. (3) Because of the recursive (sequence), we have  $a_{k+1} = 2a_k - 1.$ Because of (2), we have  $a_{k+1} = 2(2^{k}+1) - 1 = 2^{k+1} + 1$ Therefore, (3) is true.

June 11, 2025 Chapter 7 23. Let a ER st. a + 1 EZ. Prove that a"+ 1 ED for any nEN U[0]. Suratch: P(0):  $1^{\circ} + \frac{1}{4} + \frac{7}{4} + \frac{7}{4}$  $p(1) : a' + \frac{1}{a'} \in 2$  $\frac{p(2): a^2+1}{a^2} \in \mathbb{Z} \quad \text{hot so sure}$ (a' + 1) for (a2+1) - (a2+1)

We show by induction on n b o that  $a^n + \frac{1}{a^n} \in \mathbf{Z}$ . (1) for n=0, we have  $a^{\circ} + \frac{1}{a^{\circ}} = 2 \in \mathbf{Z}$ So, (1) is true for n=0. for n=1, we have  $a' + \frac{1}{a'} = a + \frac{1}{a}$ which is an integer because of the hypothesis. For n = 2, we have  $a^{2} + \frac{1}{a^{2}} = \left(a + \frac{1}{a}\right)^{2} - 2$ which is an integer because at 1 is an integer. Suppose (1) is true for all n = k for some k = 2. We show that (1) is true for n= k+1.  $a^{k+1} + \frac{1}{a^{k+1}} = \left(a^k + \frac{1}{a^k}\right) \left(a + \frac{1}{a}\right) - \left(\frac{a^k}{a} + \frac{a}{a^k}\right)$  $= \left(a^{k} + \frac{1}{a^{k}}\right)\left(a + \frac{1}{a}\right) - \left(a^{k-1} + \frac{1}{a^{k-1}}\right)$ This is an integer because  $a^k + 1$ , a + 1, a + 1 are all integers. ( de l'a) - ( de l'a) ma ( de l'a)

Chapter 9: Relation A relation on a set A is a subset of the set AXA. Here, AXA S(x,y): X & A, y & A }. in called a cartesian Product.  $max = \mu ym$ المرجوعية ومحا وجرود الشائط والأراب وا B = {1,2} AXB= { (2,1), (2,2), (3,1), (3,2), (4,1) ((4,2)} This is the confesion Product of A and B. A x A = { (2,2), (2,3), (2,4), (3,2), (3,13), (3,4), (4,2), (4,3), (4,0)} R = {(2,3), (2,4), (3,4)} CAXA. So, Rica relation: 2R3 means (2,3) ER 11-3R3 because (3,3) \$ R. 2R4 means (2,4) ER 1 xRy =7 wx, xcy, for this specific example. · x ky <=> (x,y) & R. (1940 / (p) 10) 6/13/2025 Definition: Let R be a relation on A. · R is reflexive if xRx for all XEA. A VXEA, XRX. · R is symmetric if xRy => yRx · Rin transitive if (xRy) , (yRz) => xRz. · Rh total if Yx, y EA, (xRy) v (yRx) · R is antisymmetric if (xRy) , (yRx) => X=y. · Rin dense if Yx, y EA, 37EA such that (xRZ) 1(2Ry). to on the IN , consider the following relation! man (>> gcd (min) >1. N- 51,2,3,4,5,...3 R= (6,8) (8,10) (9,21) (10,25) (4,10) (4,14) (12,18) (14,21)
R= (62,42), (42,10), (62,10), (62,10), (62,10), (62,10), (62,10)

Other I'm war. · Rin not reflexive because IEN and IRI. · Rie symmetric 1600 Ask SURIS AKE, STE Pick any mineA . We mant to show that mRm => nRm. Assume MRn. We red to show nRm. Because werbare MRn, we have g 9 Cd (min) (>1.(4.8) (1.1) (8.8) (1.8) 3 8 18/1 We know that . A dea to be about a significant that it is a significant to the significant that the significant that it is a significant to the significant that it is a significant than it is a significant that it is a significant th  $gcd(m_1n) = gcd(n_1m).$ 50 for 27, (2,2), (2,12), (3,1 gcd (nim) = gcd (min) 1210 (P. 5) (2) Therefore, mRn => nRm. 1000 1000 1000 2000 · Ru not transitive because 12 R14 and 19 R7 but 12 R7 · R unot to tal she cause = IR Mad UR 7. - (PA) man 195 Pick any min &A. We want to show that (MARCY) V (MARX). Definition, Let R. be a relation on · R is not autisymmetric because (4R6) A (GRA) but 476. · R is not dense because IRI. · Ru transing if (xRu) & inRy) => xRz. (xxx) v (noxy A Y x V y John J. 1. IN A C= (ASUM) / (ASUM) IN STATEMENTS OF A LUNG KERNING HORE AND AND AND AND SIT. in the M. coulder the feller of white .. 1 < (0,0) Ly SEX with COLOR CHAPT COLOR COLOR COLOR COLOR COLOR 3 and (and (min) comp, could forly simply - 1

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(BLA) (15)

June 14, 2025 (Math 301) Equivalence Relations An equivalence relation is a non-empty relation that is reflexive, symmetric, and transitive. Ex: A- IR R= {(a,b): a,b \in IR, a=b} ORb (=7 9=6 = { (a,a) : a & R } R is an equivalence relation. Ex: A = R R = {(9,6): a,6 (R, a 4 6)} En was from the F: Copies RE arb <=> a 6b -(US 18+10 S) + (F. U.X) + reflexive (V), transitive (V), symmetric (X) anti symmetric (V) R in (not) an equivalence relation. Ex! A= N aRb if 2 (a+b) Is R an equivalence relation? > We need to check whether R is reflexive, transitive, and symmetric. · check if R is reflexive. · check if R in symmetric. . check if R is transitive.

Check if R & reflexive Pick any 9,b € IN. We want to short aRb If and only if 2 (a+b). Assume 1, 3, 5, 7, 9, ... all "R" each other. 2, 4, 4, 8, 10, ... all "P" each other. Let R be an equivalence relation on A. For any XEA, the equivalence class of A that contains X is [x] = {yEA: xRy}. In the previous example, [1] = {1,3,5,7,9,...} [3] = {1,3,5,7,9,...} [5]: {1,3,5,7,9,...} [2] = {2,4,6,8,6,12,...} [1] 1[2] = 0 3 h a representative of the equivalence class {1,3,5,7,...}

Ex: A=N xRy (=> )= y (mod 6) (x-x=0, 6(0) · reflexive because (x-x=0/0) for any xEN. · symmetric because (x-y) le inply (y-x) le. 6/(x-4) implies 4/(y-x). [6]: {6,12,18,24,.,} [2] = {2,8,14,20,...} - [8] = [14] = ...

	June 18, 2025
	Math 301
	TOURS TONGER TOUR CHANGE Q
	$aRb = 774^2 = 2b^2 \pmod{5}$
	all (=7 2a2 + 592 = 262 (mod 5)
1	$(-7 29^2 = 26^2$
	$(=7 2a^2 - 2b^2 = 5k$
	$(=7 2(a^2-b^2)-5k$
	$(=)$ $2(a^2-b^2)=5.2l$
	(=7 a2-62 = 51
	$(-7) a^2 = b^2 \pmod{5}$
	(14 April (10) - 1 As A septil - 3
	$akb = a^2 = 6^2 \pmod{5}$
	aprivalence class
	$[0] = \{0, \pm 5, \pm \omega, \pm 1\overline{\omega}, \ldots\}$ $\alpha = 0  (\text{mod } 5)$
	[1] = {±1,±4,±6,±0, ±11,} q=±1 (mod 5)
	[2] = { ± 2, ± 3, ± 7, ± 8, 20160 ± 12, ± 13} a= ± 2 (md5)
	2
	$A = \{a \in \mathbf{I} : a \equiv 0 \pmod{5}\}$
	B= {at I: q= 1 ( nod 5) }
	C= [a { 1 : a = ±2 (mod 5)}
	To show A.O. C are all egymalouse classes:
	1) A1B1 c are distinct disjoint
	Linothing in common the Ax A is some
-	2) AUBUC = 7
	and the second of the second o
	Ap= {(x,y): x2+y2= p2 } when R > 0
	1) P= ( 11/1 / 1 / 1 / 1 / 1 / 1 / 1 / 1 / 1
	The first of the total and the first of the

*functions* 35 J. A. 160  $f(x) = x^2$  is a function.  $A = \mathbb{R}$ R= f(x,y): x,y+R, y=f(x) 3 CKxR. (akma) + 1 = 1 p = - 1 > 1 x Ry (=> y=f(x) PARTY THE THE TALL THE JUNE 1 /2 IRI 1 1 = 1 15 Fas 500 7 le and in the rej IR x for all x ≠1. 14 17 = (1, -1 3) 5 Buy A function f: A-7 A is a relation on A. x R y (=> y > f(x) (3 kg/m) = d == 0.  $R = \{(x,y) \in A \times A : y = f(x)\}$  (graph of f). all (=) a' = 6" (m: 45) Not all relations are functions. f. S1, 2, 33 = {1,2,33 { ..., 11 = , 01 ± , 0 ± R= {(1,1), (1,2), (2,3), (3,1)} } = = not a function (+ + + + ) f(1)=1 A-1062: 03 ( (mod. 5) } +(1)=2 F(2 L=2) 1 + 50 . Z = A f = A 1(2 mm) 12 18 . " = n 7 = 1 Rigorous Definition Functions A function f: A - B is a constitution of subset of A x B such that YXEA, 3! yEB St. (x,y) &f: exists uniquely A: domain of f B: co-domain of f Ex: check if the following are functions from ( + I) A= I to B-1:

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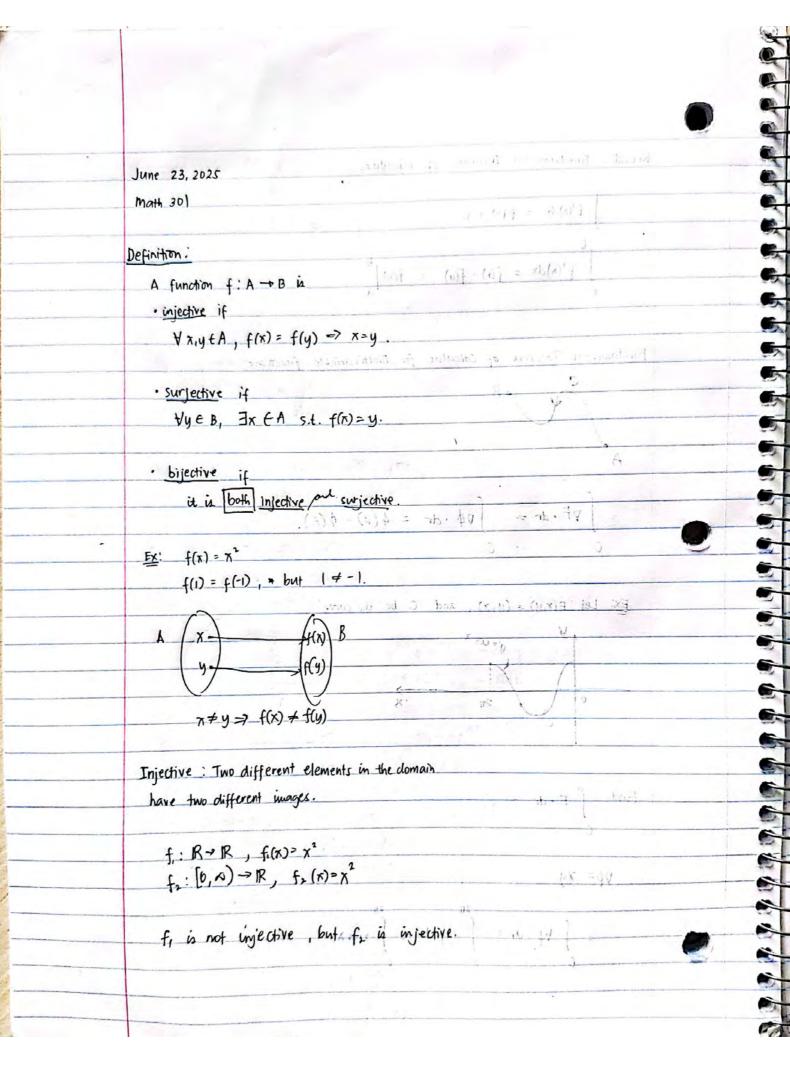
2 2 2

) f = {(n, n+1) : n∈ Z} 2) f= [(n+n): n ∈ Z} 3) f= { (n+1, n) : n = 2} 4) f= {(n,1/n) : n ∈ I} Negation: FREA S. t. either there is no y such that (Kiy) & f or there are more than one such y. 1) f= {(nin+1) : n ∈ I} Notation: AxEA st. (YyEB, (x,y) & f) V (Ay, ZEB s.t. Yx EA, 3! y EB st. (x,y) Ef. y # 7 (x,y) Ef 1 (n, 2) +f). Pick any x = I We want to find y = I such that (X,y) & frond show that such y is unique. Chose/Pick y= x+1. We have (x,y)=(x,x+1) &f. Suppose 26 1 and (x, 2) & f. Then 2 = x+1 = y . So, y in mique. 2.) f = \( (n^2, n) : n \in \mathbb{Z} \right\) Choose N=-1. We show that for any yEB, (x,y) &f. Pick any yEZ. We want to show (x,y) & f. Suppose by contradiction that (x,y) &f. Thun (x,y) ~ (n2,4) for some n & t. (-1=42) contradiction.

1 A 1 D f = {(n+1, n) : n∈ I}. Pick any XEI. We want to find and show that y is unique. Choose y=x-1. we have (x,y) = (x,x-1) = (x+1,x) & f. Suppose ZE I and (x, Z) Ef. Then 7 = x-1 > y. So, y is inique. (A) f = {(n, 1/4): ne I} we want to show that for any yt II, (x,y) & f. Pick any y∈ 7 We want to show (x, y) €f. Suppose, by contradiction, that (x,y) Ef. Them (x,y) = (n, n2+1) for some h = 1. So, for x = h=1, we have This is a confindration because ; ET. Transforme, there is no souch y.

June 20, 2025 Function f: A - B in a subset of AXB such that: YxfA, 3!yEB s.t. (x,y) Ef. (\*) (x) is equivalent to: 1) YREA, JyEB s.t. (x,y) Ef. 2) YXEA , y, Z EB, (X, y) &f 1 (X, 7) &f => y=2. (\*) (=> (1) N(2) Notation: Let f is a function from A to B. If (xiy) Ef then we write grif(x) and (the context) We say that y is the image of x under f.

Ext f={(1,2),(2,3),(3,0)} f(1)=2 : 2 is the image of 1 under f. f(0) te undefined. Let f: A - B be a function. The set of all images under f is called the range of f. range (f) = { y EB : = x EA s.t. y = f(x)}. = {f(x): x ∈ A} Another notation:  $f(A) = \{f(x) : x \in A\}$ Ex: f= ((1,2), (2,3), (3,0)} range (f): 5(2), 3, 0}. dorain (f) = {1,2,3}.



## Horizontal Line Test:

$$f(x_1) = f(x_2) = f(x_3) = f(x_4)$$
  
f is not injective

The function  $f: A \rightarrow B$  (where A,B are subsets of R)

is injective if and only if any horizontal line intersects

the graph of f at at most one point.

to my to proper with

Ex: show that 
$$f: (1, A) \rightarrow \mathbb{R}$$
,  

$$f(x) = \frac{2x+1}{x-1}$$

Pick any 
$$x_1y \in (1/\infty)$$
. We want to show that
$$f(x) = f(y) \implies x = y.$$

$$\frac{2x+1}{x-1} = \frac{2y+1}{y-1} \implies x=y. \tag{1}$$

Assume that the LHs of (U is true, We have,

$$\frac{2x+1}{x-1} = \frac{2y+1}{y-1}$$

By multiplying each side by the product of their denominators, we get

$$(2x+1)(y-1) = (2y+1)(x-1)$$
 which is again alent to  $2xy+y-2x-1 = 2yx+x-2y-1$ . (2)

By simplifying (2), we get

$$\begin{bmatrix} 2y + y = -9x \pm x, \\ 3y = -9x, \\ y = x, \end{bmatrix}$$

$$\begin{bmatrix} 3x = 3y, \\ x = y. \end{bmatrix}$$

List was promised by By simplifying (2), ne get 3x = 3y which implies x= y . n (pr) = (ch) = (ch) = (M) Surjective art joi tou unit B ( to per the sun to the sound of a-A : 1 - receive All terrain and laterative place in place than it was ever tring and town it its fig to price and f: A -> B is surjective if every yEB , she (a): , fort work -35 is the image of some XEA. 赵 f: R→R, f(x) = x2 f is not surjective because there is no XER such that f(X)=-1. Rich and rule Copa, We want to strong that 18 W 18 X Ex:  $f: R \rightarrow [0,\infty)$ ,  $f(K) = \chi^2$ . Show that f is surjective. Pick any JER. We will show that there exists y (Fax) such that y f(x)=x2. We need to find you for such that y ff(x)=x2. erry , and the property of the party of the second property of Choose X= Ty. We have, (1-x)(111) -(1-4)(11x4) f(1)= (1)2 which to equivalent to fale 1. The way I compression is y = (14)2 which is equivalent to y = y - a ....

10. Let f: I - IXI be defined by f(n) = (2n+1, n+2). Check whether this function is injective and whether it is surjective, prove your answer. Injective. Pick any x, y & I. We want to show that (2x+1, x+2) = (2y+1, y+2) = 7 x = y. (1) Assume His of to in true we have [2x+=2y+1 ] x+2=y+2. (2) [tec 2x+1=2g+1, by simplifying we will get] by simplifuging (2), no get Assume LHS of (1) is true. We have X+2-y+2, which implies X= y. .

f:1R/[1] -> R/ [2], 12 bijective. · check if it is injective. Pick any my ( ) [R \ {1} [ Cod sy ( 12 \ 23 )] We want to show that Assume 2x+1 = 2y+1 => x=y (1). We went to show that X=Yby multiplying each ride by the product of their denominators, we get us (2x+1) (y-1)=(2y+1)(x-1), which is equivalent to 2 xy +y-2x-1= 7yx+x-2y-1. (7) By suplifying (2), we get 3y = 3x which implies X = 4 Therefore, of is unjective. , check if its is surjective. Pick any. y & R \ {23. We want to show that JX (17) {13 such that ext = y. We will find x ETR \ {13 such that 2x+1=y. (7) Pick X= 2. We get  $\frac{2(2)+1}{2-1}=y, \text{ which is equivalent to}$ 5 = 4. Since 5 + 12/23, we have shown that the function in surjective.

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Pick x = -y-1. We Grates have topual if 2-4 1: ( ) 1 1 5 5 5 5 6 Equivalet if . Distilled fi sunther i egual.  $f(x) = \frac{2\left(-y-1\right)}{2-y} + 1 \text{ (body)} \qquad (1)$ The RHK of (1) can be simplified to : A 1 1 h Cyty A wild (2 110 , 111) - 11 -2y-2+2-4 register from 12-142) big 1. which (P. in) Equal to 11 -y-1-2+y 12 11 11 11 11 2-y strong eghal super which is equivalent to Smee f(x)=y, we have shown that Therefore, the function is bijective.

- 6.) Consider the following relation on the set A =. ShEN: n≥23.

  art => ged (a,b)=1
  - because gcd(3,3) = 3, He had 3\$\mathread{\mathread{Z}}.

    So, It is not reflexive.
  - · Check if symmetric.

    Pick, a, b & A . Ne wart to show that

    my Va,b & A . arb => b Ra.

    Assume ARb. We need to show bRa.

    Because ARb , ne have

 $gcd(n_1b) = 1$ .

No know that  $gcd(b_1a) = gcd(a_1b) = 1$ .

Therefore, allb => bka.

- \* Check if transitive.

  Because gcd (5,9)=1 and gcd (9,10)=1, but gcd (5,10) \$\vec{x}\$ 5\$,

  We have 5\$\vec{x}\$ to: 5kg and 9 klo but 5 klo.

  Therefore, R is not transitive.
- Because 3KG and 6K3, Rie not total.
- · Chert if asymmetric.

  Because 2R3 and 3R2 but 2+3, R is not asymmetric.
- · Check if dense.

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