Worksheet – Sections 5.3

Conditional probability: $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$ (probability of A given B)

Independent events: two events are independent if the occurrence or non-occurrence of one event does not affect the probability of the other event. In that case, the following are equivalent:

$$P(A|B) = P(A), \qquad P(B|A) = P(B), \qquad P(A \text{ and } B) = P(A)P(B).$$

Associated events: two events are called associated (or dependent) events if they are not independent.

1) Suppose you roll two fair dice. In each of the following scenarios, write the sample space and compute the probability.

a) What is the probability that the two numbers on the top faces of the dice are the same?

b) What is the conditional probability that the two numbers are the same, given that the sum of these numbers is 6?

c) What is the conditional probability that the two numbers are the same, given that the sum of these numbers is 7?

2) Suppose you flip a coin 4 times. In each of the following scenarios, write the sample space and compute the probability.

a) What is the probability of getting all Heads?

b) What is the probability of getting all Heads, given that the first 3 flips were Heads.

3) Suppose you withdraw a card from a standard deck of 52 cards.

a) Find the probability of getting a spade.

b) Find the probability of getting a black card.

c) Find the probability of getting a spade, given that the card is black.

d) Are the events “the card is a spade” and “the card is black” independent or associated events?

4) A playing card is dealt facedown. You are interested in knowing whether our opponent holds an ace. Suppose that you have discovered that his card is a diamond. Is this helpful information?

5) You tossed an unfair coin with $P(Head) = 3/4$ and $P(Tail) = 1/4$. If Head comes up, you draw a marble from a jar containing 4 green marbles and 4 yellow marbles. If Tail comes up, you draw a marble from a jar containing 5 green marbles and 10 yellow marbles.

a) Find the probability that getting a Head and drawing a yellow marble.

b) Are the events “Head shows up” and “Yellow marble is drawn” independent or associated events?

c) Find the probability that getting a yellow marble.