

Worksheet – Sections 7.3-7.4

Central Limit Theorem: supposing that X_1, X_2, \dots, X_n are independent random variables with the same distribution. Then for large n , the average $\frac{X_1 + X_2 + \dots + X_n}{n}$ is approximately normally distributed with mean $= \mu_{X_1}$ and standard deviation $= \frac{\sigma_{X_1}}{\sqrt{n}}$.

Sample proportion \hat{p} is approximately normally distributed with mean $= p$ and standard deviation $= \sqrt{\frac{p(1-p)}{n}}$ if the **conditions for Central Limit Theorem for Sample Proportions** are met:

- Simple random sampling: every group of n individuals in the population has an equal chance of being selected
- Large sample size: $np > 10$ and $n(1 - p) > 10$
- Large population: $N > 10n$

$x\%$ confidence interval: $[\hat{p} - a, \hat{p} + a]$ where $P(p \in [\hat{p} - a, \hat{p} + a]) = x\%$. The number a is called the **margin of error** (often expressed in percentage points).

1) A university states that 42% of all students own a tablet. You take a random sample of 150 students from a university with 20,000 students.

(a) Comment on whether the conditions for CLT for Sample Proportions can be met.

(b) Assuming that these conditions hold, find the mean and standard deviation of the sample proportion.

(c) You expect _____% of students in your sample to own a tablet, give or take _____ percentage points.

(d) Find the probability that the sample proportion is less than 35%.

2) A company reports that 28% of employees work remotely. You select a random sample of 100 employees from a workforce of 2,500 employees.

(a) Comment on whether the conditions for CLT for Sample Proportions can be met.

(b) Assuming that these conditions hold, find the standard error.

(c) You expect _____% of employees in your sample to work remotely, give or take _____ percentage points.

(d) Find the probability that the sample proportion is greater than 40%.

3) A researcher takes a random sample of 400 households from 1.2 million households in a state. He/she finds that 72 households have solar panels.

(a) Find the sample proportion.

(b) Find an estimate for standard error.

(c) Comment on whether the conditions for CLT for Sample Proportions can be met.

(d) Assuming that these conditions hold, find the 95% confidence interval for the proportion of all households in the state who have solar panels. Find the margin of error.

(e) Find the 90% confidence interval for the proportion of all households in the state who have solar panels. Find the margin of error.