Worksheet – Sections 7.5

Theorem: Let X_1 and X_2 be two independent random variables. Suppose that X_1 is normally distributed with mean μ_1, standard deviation σ_1, and that X_2 is normally distributed with mean μ_2, standard deviation σ_2. Then $X_1 \pm X_2$ is normally distributed with mean $\mu_1 \pm \mu_2$ and standard deviation $\sqrt{\sigma_1^2 + \sigma_2^2}$.

Application: Consider two populations. Let p_1 and p_2 be the proportions of two populations that have certain qualities. Random samples of size n_1 and n_2 are drawn from respective populations, resulting in sample proportions $\hat{p}_1$ and $\hat{p}_2$. If the conditions listed below are satisfied, then the difference $\hat{p}_1 - \hat{p}_2$ is approximately normally distributed with mean $p_1 - p_2$ and standard deviation

$$\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

- Simple random sampling is used for each population
- Large sample size condition is satisfied in each population
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- Independence between samples: the selection of individuals for one sample does not affect the selection of individuals for the other sample, and the outcomes of one sample do not affect or predict the outcomes of the other.

***x*% two-proportion confidence interval** for the difference $p = p_1 - p_2$ is $[\hat{p} - a, \hat{p} + a]$ where $\hat{p} = \hat{p}_1 - \hat{p}_2$ and $P(p \in [\hat{p} - a, \hat{p} + a]) = x\%$.

1) A national survey found that 87% of teenagers own a smartphone. The survey's margin of error was ±2 percentage points with a 95% confidence level.

(a) State the survey results in confidence interval form and interpret the interval.

(b) If 300 similar surveys were conducted in the nation, how many of them would result in confidence intervals that included the true population proportion?

(c) A student wrote this interpretation of the confidence interval: "We are 95% confident that the sample proportion is within the confidence interval." What is incorrect about this interpretation and how would you fix it?

2) A technology company wants to compare customer satisfaction between two versions of its mobile app. A random sample of users of each version was surveyed and asked whether they were satisfied with the app.

	Satisfied	Not satisfied
Version A	168	72
Version B	151	89

(a) Find the percentage of satisfied customers for each version. Which version has a higher percentage of satisfied customers?

(b) Comment on whether the conditions for a two-proportion confidence interval are satisfied.

(c) Find the estimate mean and standard deviation for the proportion difference $p_1 - p_2$.

(d) Find the 90% confidence interval for the difference in the proportions of each group. Interpret the meaning of this interval.

(e) Find the 95% confidence interval for the difference in the proportions of each group. Interpret the meaning of this interval.

3) A school district of 4589 students randomly assigned struggling students to either receive after-school tutoring or not receive tutoring. At the end of the year, students were classified by to whether they passed a math exam.

	Pass	Fail
Tutored	60	40
Not tutored	45	55

(a) Find the percentages that pass the exam for both groups. Was tutoring associated with a higher passing rate?

(b) Comment on whether the conditions for a two-proportion confidence interval are satisfied.

(c) Find the estimate mean and standard deviation for the proportion difference $p_1 - p_2$.

(d) Find the 95% confidence interval for the difference in the proportions of each group. Interpret the meaning of this interval.

4) Determine if the sampling of the two populations is reasonably independent.

(a) A researcher wants to compare the proportion of students of BYU-Hawaii with that of University of Hawaii who own a bicycle. He randomly samples 100 students from each university.

(b) A researcher wants to compare the proportion of males with the proportion of females who exercise regularly. He surveys 100 married couples and records whether the husband exercises regularly, and whether the wife exercises regularly.

(c) A researcher wants to compare the proportion of people owning a tablet with the proportion of people owning a smart phone. He surveys 100 people and records whether they own a tablet or smart phone.

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
− 3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
− 2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
− 2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
− 2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
− 2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
− 2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
− 2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
− 2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
− 2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
− 2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
− 2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
− 1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
− 1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
− 1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
− 1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
− 1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
− 1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
− 1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
− 1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
− 1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
− 1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
− 0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
− 0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
− 0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
− 0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
− 0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
− 0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
− 0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
− 0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
− 0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
− 0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641