Homework 1: solution to Problem 5 and 38, page 295–296

5) $z = 3 - 5i$

$w = 2 + 7i$

$z + w = 3 + 2 + (-5+7)i = 5 + 2i$

$zw = (3-5i)(2+7i) = 6 - 10i + 21i - 35i^2 = 41 + 11i$

$z^2 = (3-5i)^2 = (3-5i)(3-5i) = 9 - 15i - 15i + 25i^2 = -16 - 30i$

$\bar{z} = 3 + 5i$

$z\bar{z} = (3-5i)(3+5i) = 9 - 25i^2 = 9 + 25 = 34$

$(\bar{z})^2 = (3+5i)^2 = (3+5i)(3+5i) = 9 + 15i + 15i + 25i^2 = -16 + 30i$

$$\frac{1}{z} = \frac{\bar{z}}{z\bar{z}} = \frac{\bar{z}}{|z|^2} = \frac{3+5i}{3^2+(-5)^2} = \frac{3+5i}{34} = \frac{3}{34} + \frac{5}{34}i$$

$$\frac{w}{z} = w \cdot \frac{1}{z} = (2+7i)\left(\frac{3}{34} + \frac{5}{34}i\right) = \frac{6}{34} + \frac{21}{34}i + \frac{10}{34}i + \frac{35}{34}i^2$$

$$= \frac{6}{34} + \frac{31}{34}i - \frac{35}{34}$$

$$= \frac{-29}{34} + \frac{31}{34}i$$

$$\frac{z}{w} = \frac{3-5i}{2+7i} = \frac{(3-5i)(2-7i)}{(2+7i)(2-7i)} = \frac{6 - 10i - 21i + 35i^2}{2^2 + 7^2} = \frac{-29 - 31i}{53}$$

$$= \frac{-29}{53} - \frac{31}{53}i$$

38) $f(x) = 2x^4 - 7x^3 + 14x^2 - 15x + 6$

First, notice that $f(1) = 2 - 7 + 14 - 15 + 6 = 0$, so $x = 1$ is a root. That means $x - 1$ is a factor of $f(x)$. Use synthetic division:

1	2	-7	14	-15	6
		2	-5	9	-6
	2	-5	9	-6	0

$f(x) = (x-1)(\underbrace{2x^3 - 5x^2 + 9x - 6}_{g(x)})$

Note that $g(1) = 2 - 5 + 9 - 6 = 0$. So, $x-1$ is a factor of $g(x)$.

Synthetic division:

$$\begin{array}{c|cccc} 1 & 2 & -5 & 9 & -6 \\ & & 2 & -3 & 6 \\ \hline & 2 & -3 & 6 & 0 \end{array} \qquad g(x) = (x-1)(2x^2-3x+6)$$

Therefore, $f(x) = (x-1)g(x) = (x-1)^2(2x^2-3x+6)$.

We can find the roots of $2x^2-3x+6$ using the quadratic formula:

$$x = \frac{3 \pm \sqrt{(-3)^2-4(2)(6)}}{2(2)} = \frac{3 \pm \sqrt{-39}}{4} = \frac{3 \pm i\sqrt{39}}{4}$$

Therefore,

$$f(x) = 2(x-1)^2\left(x - \frac{3-i\sqrt{39}}{4}\right)\left(x - \frac{3+i\sqrt{39}}{4}\right)$$

The roots of $f(x)$ are 1, $\frac{3-i\sqrt{39}}{4}$, and $\frac{3+i\sqrt{39}}{4}$.