

HW10 solution (Prob 32)

Monday, March 20, 2023 9:26 PM

Solve the equation $e^x + 15e^{-x} = 8$.

Let $t = e^x > 0$. Then $e^{-x} = \frac{1}{e^x} = \frac{1}{t}$.

The original equation can be rewritten as

$$t + 15 \frac{1}{t} = 8.$$

Multiply both sides by t : $t^2 + 15 = 8t$

$$\leadsto t^2 - 8t + 15 = 0$$

$$\leadsto (t-3)(t-5) = 0$$

$$\leadsto t = 3 \text{ or } t = 5$$

$$\leadsto e^x = 3 \text{ or } e^x = 5$$

$$\leadsto x = \ln 3 \text{ or } \ln 5$$

Conclusion: the equation has two solutions: $\ln 3$ and $\ln 5$.