Lab 1

In this lab, we will warm up with Mathematica and practice with Riemann sums.

1 Get access

You can find the instruction to install Mathematica on your computer for free here:

https://web.engr.oregonstate.edu/~phamt3/Resource/Wolfram-Mathematica-with-JupyterLab.pdf

Alternatively (and more simply), you can use the cloud-based version of Mathematica* here:

https://www.wolframcloud.com

If you do not have a Wolfram account, please create one. Watch the following video to get started:

https://youtu.be/5wsfG80oD1g

2 First experiments

(1) Type `35/6`, then `Shift+Enter`.

(2) Type `N[35/6]` (notice the square brackets), then `Shift+Enter`.

(3) Type `Sqrt[2]` (notice the capitalized S), then `Shift+Enter`.

(4) Type `N[%]`, then `Shift+Enter`.

(5) Type `Sin[Pi]` (notice the capitalized S and P), then `Shift Enter`.

(6) Type `34^100;` (with the semicolon), then `Shift Enter`.

(7) Type `34^100` (without semicolon), then `Shift Enter`.

You may have noticed that the function `N` is to evaluate a numerical value of an expression. Each function's name is capitalized and used with square brackets (not with parentheses as we usually write). The semicolon is to hold the output. One uses it when output is too long or not of interest. Next, try the following:

(8) `Exp[1]`, then `Shift+Enter`.

(9) `Log[2]`, then `Shift+Enter`.

(10) `f[x_] := Sin[x]+Cos[x]` (notice the dash after `x`), then `Shift+Enter`.

(11) `f[Pi]+f[Pi/4]`, then `Shift+Enter`.

(12) `Clear[f]`, then `Shift+Enter`.

(13) `f[Pi]+f[Pi/4]`, then `Shift+Enter`.

The natural logarithm function is named `Log` in Mathematica (not `ln`). `Exp` is the exponential function. Command (10) is to define a function. The dash is required in order to tell Mathematica that we are defining the function f. The function `Clear` is to remove a defined variable from the memory.

*limited to about 8 minutes of computation per month. Files will be deleted from cloud storage after 60 days.

3 Plot the graph of a function

First, let us plot functions of one variable, for example the sine function `sin(x)`. Try the following commands:

(14) `Plot[Sin[x], {x,0,2*Pi}]`, then `Shift+Enter`.

(15) For decoration, try

```
Plot[Sin[x], {x,0,2*Pi}, PlotStyle -> {Red, Dashed}]
```

Then `Shift+Enter`. Note that the arrow is typed as $->$.

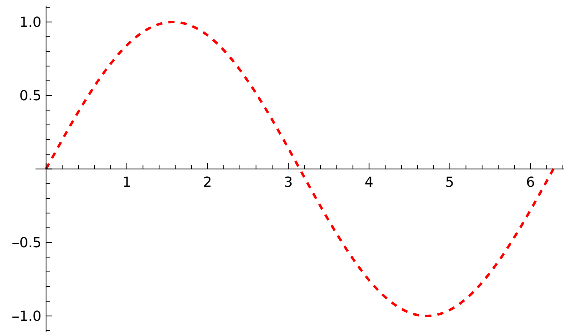

Figure 1

(16) You can also give the function a name before plotting it. For example,

```
f[x_] := Sin[x];
Plot[f[x], {x,0,2*Pi}, Filling→Axis]
```

Then `Shift+Enter`. Note that the dash following x within the brackets is no longer used because f was already defined.

4 Compute Riemann sums

(17) To compute $1 + 2 + 3 + ... + 100$, we write this sum in sigma notation as $\sum_{k=1}^{100} k$. We can evaluate this formula with the command:

```
Sum[k,{k,1,100}]
```

(18) To compute $2^2 - 3^2 + 4^2 - 5^2 + ... - 99^2 + 100^2$, we write this sum in sigma notation as $\sum_{k=2}^{100}(-1)^{k+1}k^2$. We can evaluate this formula with the command:

```
Sum[(-1)^(k+1)*k^2,{k,2,100}]
```

(19) Now let us evaluate the area under the parabola $f(x) = x^2$ on the interval $[1, 2]$ (Figure 2).

```
Plot[x^2,{x,1,2},Filling->Axis, PlotRange->{{0,3},{0,5}}]
```

If we divide the interval $[1, 2]$ into n equal subintervals, then the grid-points are $x_0 = 1$, $x_1 = 1 + \frac{1}{n}$, $x_2 = 1 + \frac{2}{n}$,..., $x_n = 1 + \frac{n}{n} = 2$. In general, $x_k = 1 + \frac{k}{n}$ for any number k between 0 and n. Recall the Riemann sums:

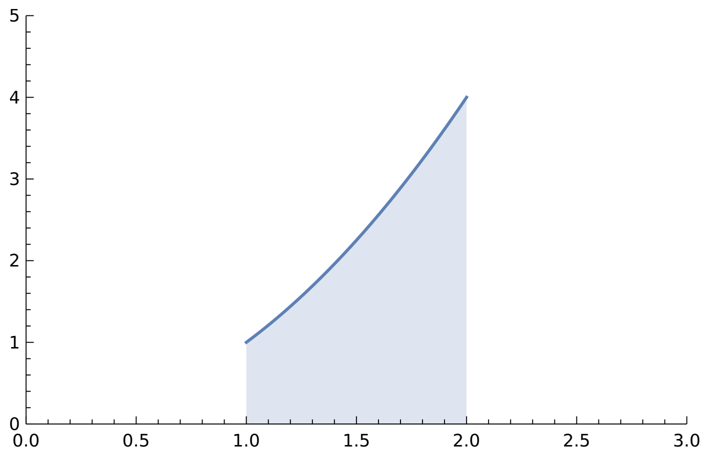

Figure 2

- Left-point rule:

$$L_n = \sum_{k=0}^{n-1} f(x_k)\Delta x$$

- Right-point rule:

$$R_n = \sum_{k=1}^{n} f(x_k)\Delta x$$

- Mid-point rule:

$$M_n = \sum_{k=0}^{n-1} f\left(\frac{x_k + x_{k+1}}{2}\right)\Delta x$$

- Trapezoid rule:

$$T_n = \sum_{k=0}^{n-1} \frac{f(x_k) + f(x_{k+1})}{2}\Delta x$$

To compute L_{1000}, we can write:

```
n=1000;
dx=(2-1)/n;
f[x_]:=x^2;
Sum[f[1+k/n]*dx,{k,0,n-1}]
```

Can you write commands to find R_{1000}, M_{1000}, and T_{1000} ?

(20) To find the true value of the area, we need to take the limit of one of the Riemann sums (L_n, R_n, M_n, T_n) as $n \to \infty$.

```
Clear[n];
dx=(2-1)/n;
L[n_]:=Sum[f[1+k/n]*dx,{k,0,n-1}];
Limit[L[n],n->Infinity]
```

You will get $\frac{7}{3}$. Which of those four Riemann sums is the best estimate for the true value of the area? Which one is the worst?

(21) Can you adjust the procedure above to evaluate the exact area under the curve $y = 1/x$ when $x \in [2, 5]$?

5 To turn in

Submit your implementation of Exercises (1) - (21) as a single pdf file.