Lab 3

In this lab, we will practice with Mathematica the following topics:

- Find the limits and derivatives of a function.
- Visualize exponential, inverse trigonometric, and inverse hyperbolic trigonometric functions.
- Compute and visualize inverse functions.

1 Reminder about getting access

There are two ways to get free access to Mathematica:

A) Install three free components: *Wolfram Engine*, *JupyterLab*, and *WolframLanguageForJupyter*. You can use the unlimited computing power of Mathematica on your own computer, with Jupyter Notebook acting as a user interface. The instruction is here:

https://web.engr.oregonstate.edu/~phamt3/Resource/Wolfram-Mathematica-with-JupyterLab.pdf

B) Use the cloud-based version of Mathematica: https://www.wolframcloud.com
In this option, you are limited to about 8 minutes of computation per month. Files stored on the cloud will be deleted after 60 days.

2 Find limits

(1) Type `Limit[(E^(x^2)-1)/x^2, x->0]`, then `Shift+Enter`.

(2) Type `a = Limit[(1+1/x)^x, x->Infinity]`, then `Shift+Enter`.
Type `N[a,6]`, then `Shift+Enter`.
Type `N[a,10]`, then `Shift+Enter`.
Type `N[a,15]`, then `Shift+Enter`.

(3) Use Mathematica to do Problems 10 of Section 5.8 on page 311 of the textbook. Round the result to 4 decimal places.

(4) Use Mathematica to do Problems 12 of Section 5.8. Round the result to 4 decimal places.

(5) Use Mathematica to do Problems 33 of Section 5.8. Round the result to 4 decimal places.

(6) Use Mathematica to do Problems 34 of Section 5.8. Round the result to 4 decimal places.

3 Find derivatives

(7) Type `f[x_]:=Cos[x]`, then `Shift+Enter`.
Type `f'[x]`, then `Shift+Enter`.
Type `f'''[x]`, then `Shift+Enter`.

(8) Graph the function $f(x) = a^x$ on the interval $[-2, 2]$ with $a = \frac{1}{6}, \frac{1}{3}, \frac{1}{2}, 1, 2, 3, 6$. What do you observe about the increasing/ decreasing behaviors of those functions for each value of a ?

(9) For the above values of a, find the slope of the curve at $x = 0$. In other words, find $f'(0)$. What is the slope of the curve at $x = 0$ when $a = e$?

(10) Find the derivative of the function $f(x) = x^x$.

4 Optimization problem

(11) Graph the function $f(x) = x^x$ on the intervals $[0, 1]$, $[1, 3]$, and $[0, 3]$. Does this function have a minimum value and/or maximum value for $x > 0$?

(12) Find all the critical numbers of f.

(13) Find the minimum value of f and the value of x where the minimum occurs. Express those values precisely and also numerically (4 decimal places).

5 The inverse trigonometric functions

The inverse sine, cosine, tangent, cotangent in Mathematica are **ArcSin, ArcCos, ArcTan, ArcCot**. The inverse hyperbolic sine, cosine, tangent, cotangent in Mathematica are **ArcSinh, ArcCosh, ArcTanh, ArcCoth**.

(14) Plot the function $f(x) = \arcsin(x)$ and determine from the graph the domain, range, asymptotic behaviors of this function.

(15) Plot the function $f(x) = \arccos(x)$ and determine from the graph the domain, range, asymptotic behaviors of this function.

(16) Plot the function $f(x) = \arctan(x)$ and determine from the graph the domain, range, asymptotic behaviors of this function.

(17) Plot the function $f(x) = \text{arccot}(x)$ and determine from the graph the domain, range, asymptotic behaviors of this function.

(18) Plot the function $f(x) = \text{arcsinh}(x)$ and determine from the graph the domain, range, asymptotic behaviors of this function.

(19) Plot the function $f(x) = \text{arccosh}(x)$ and determine from the graph the domain, range, asymptotic behaviors of this function.

(20) Plot the function $f(x) = \text{arctanh}(x)$ and determine from the graph the domain, range, asymptotic behaviors of this function.

(21) Plot the function $f(x) = \text{arccoth}(x)$ and determine from the graph the domain, range, asymptotic behaviors of this function.

6 Compute and visualize inverse functions

(22) Let $f(x) = \frac{2x+1}{3x-2}$. To find the inverse of this function, we set $x = \frac{2y+1}{3y-2}$ and solve for y. Try

```
Solve[x==(2y+1)/(3y-1),y]
```

What do you get for $f^{-1}(x)$?

(23) The above problem can be solved by hand. However, the following problem is much harder to do by hand. Let $f(x) = x^3 + x$. Use derivative to explain why it has an inverse.

(24) Then find the inverse using the command **Solve** shown above. (Note that Mathematica will give three solutions, two of which are complex valued. You should only take the real valued expression.)

7 To turn in

Submit your implementation of Exercises (1) - (24) as a single pdf file.