# Lab 4

In this lab, we will practice with Mathematica the following topics:

- Factoring and expanding a polynomial.
- Finding the real and complex roots of a polynomial.
- Finding the quotient and remainder of a polynomial division.
- Partial fraction decomposition.
- Evaluating the definite or indefinite integral of a function.

## 1 Reminder about getting access

There are two ways to get free access to Mathematica:

A) Install three free components: *Wolfram Engine*, *JupyterLab*, and *WolframLanguageForJupyter*. You can use the unlimited computing power of Mathematica on your own computer, with Jupyter Notebook acting as a user interface. The instruction is here:

https://web.engr.oregonstate.edu/∼phamt3/Resource/Wolfram-Mathematica-with-JupyterLab.pdf

B) Use the cloud-based version of Mathematica: https://www.wolframcloud.com
In this option, you are limited to about 8 minutes of computation per month. Files stored on the cloud will be deleted after 60 days.

## 2 Factor and expand a polynomial

(1) Type `Expand[(1+x)^5*(2-x)^4]` then `Shift+Enter`.

(2) Now try the command

```
TraditionalForm[Expand[(1+x)^5*(2-x)^4]]
```

What is the difference do you see in the output compared to the output of the previous command?

(3) As a syntax rule of Mathematica, writing `expr1[expr2]` is equivalent to writing `expr2 // expr1`. The second way is free of square brackets and is usually more convenient than the first way. Therefore, the command in the previous exercise can be written as

```
(1+x)^5*(2-x)^4 // Expand // TraditionalForm
```

(4) Expand the polynomial $f(x) = (2x^2 + x + 1)^7(x - 1)^3$ and arrange the terms in descending powers. What is the degree of $f$ ?

(5) Try the commands

```
Factor[x^3+x^2-2]
x^3+x^2-2//Factor
x^3+x^2-2//Factor//TraditionalForm
```

(6) Determine all the real roots together with their multiplicities of the polynomial

$$f(x) = x^7 - 6x^6 + 11x^5 - 22x^3 + 20x^2 + 8x - 16$$

(7) To simplify the rational function $\dfrac{x^3 + x^2 - 2}{x^2 - 3x + 2}$, use one of the commands

```
Simplify[(x^3+x^2-2)/(x^2-3x+2)]
Simplify[(x^3+x^2-2)/(x^2-3x+2)]//TraditionalForm
```

(8) In Exercise 5, simplify the quotient $\frac{f'(x)}{f(x)}$. If you forget how to take the derivative of a function using Mathematica, refer to Lab 3, Section 3.

# 3 Find real and complex roots

(9) You can find roots of a polynomial by factoring it as in Exercise 6. Alternatively, you can use the command `Solve`. Try the following:

```
f[x_]:=4x^9- 12x^8+ 9x^7- 42x^6+ 120x^5- 42x^4- 104x^3+ 48x^2+ 21x- 10
          Solve[f[x]==0,x]
```

(10) What are the multiplicity of each root in the previous exercise ?

(11) To find the real/rational/integer roots only, add the option `Reals/Rationals/Integers`:

```
Solve[f[x]==0,x,Reals]
Solve[f[x]==0,x,Rationals]
Solve[f[x]==0,x,Integers]
```

(12) To find roots as numerical values, we use the command `NSolve`. Try the following:

```
NSolve[(x-1)^3==1/x,x]
```

# 4 Find the quotient and remainder of a polynomial division

(13) Let $f(x)$ and $g(x)$ be two polynomials. You can find the quotient and remainder of the division $f(x) \div g(x)$ by hand using long division. If $q(x)$ is the quotient and $r(x)$ is the remainder then $f(x) = g(x)q(x) + r(x)$. On Mathematica, you can find $q(x)$ and $r(x)$ via the command `PolynomialQuotientRemainder`. The syntax is

```
PolynomialQuotientRemainder[f(x),g(x),x]
```

The output is `{q(x),r(x)}`. Try the following:

```
PolynomialQuotientRemainder[x^4 + 2x + 1, x^2 + 1, x]
PolynomialQuotientRemainder[x^4 + 2x + 1, x^2 + 1, x]//TraditionalForm
```

(14) Find the quotient and remainder of the division $(x^6 - 1) \div (x - 2)$. Then use this result to find the integral

$$\int \frac{x^6 - 1}{x - 2} dx$$

## 5 Partial fraction decomposition

The command `Apart` decomposes a rational function into partial fractions.

(15) For example, the decompose the function

$$f(x) = \frac{x^4 - 1}{(x-2)(x-3)}$$

into partial fractions, try the following:

```
Apart[(x^4-1)/((x-2)*(x-3))]
Apart[(x^4-1)/((x-2)*(x-3))]//TraditionalForm
```

(16) Do Problem 3a of Section 6.3 (page 340).

(17) Do Problem 3b of Section 6.3 (page 340).

(18) Do Problem 4a of Section 6.3 (page 340).

(19) Do Problem 4b of Section 6.3 (page 340).

## 6 Find integrals

The command `Integrate` is used to find definite and indefinite integrals. The command `NIntegrate` is used to find a numerical approximation of a definite integral. The syntax is as follows:

- `Integrate[f(x),x]` for $\int f(x)dx$
- `Integrate[f(x),{x,a,b}]` for the exact value of $\int_a^b f(x)dx$
- `NIntegrate[f(x),{x,a,b}]` for a numerical approximation of $\int_a^b f(x)dx$

(20) To find $\int \frac{1}{x}dx$, try the following

```
Integrate[1/x,x]
```

(21) To find $\int_{-2}^{-1} \frac{1}{x}dx$, try the following

```
Integrate[1/x,{x,-2,-1}]
NIntegrate[1/x,{x,-2,-1}]
```

Note: in Mathematica, Log[$x$] means $\ln(x)$.

(22) Find the exact and numerical values of the definite integral in Problem 8 of Section 6.2 (page 332).

(23) Find the exact and numerical values of the definite integral in Problem 42 of Section 6.2 (page 332).

(24) Find the indefinite integral in Problem 14 of Section 6.3 (page 340).

(25) Find the indefinite integral in Problem 4 of Section 6.4 (page 346).

## 7 To turn in

Submit your implementation of Exercises (1) - (25) as a single pdf file.