Lecture 11

Friday, January 27, 2023 10:37 AM

* Question

Fundamental thm of Calc

$$\int_a^b f(x)\,dx = F(b) - F(a),$$ where F is any antiderivative of f.

How do we find antiderivatives?

Simple antiderivatives

$$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1} + C \qquad (\alpha \neq -1)$$

$$\int \sin(\alpha x)\,dx = -\frac{1}{\alpha}\cos(\alpha x) + C$$

$$\int \cos(\alpha x)\,dx = \frac{1}{\alpha}\sin(\alpha x) + C$$

Two major methods of finding antiderivatives we will learn in this course are

* Substitution method
* Integration by parts

<u>Substution method</u> (informally called "u-substitution)

<u>Ex</u> $$\int x\sqrt{1+x^2}\,dx$$

Recall. $y = f(x)$

$$\underbrace{dy}_{\text{change in } y} = f'(x)\underbrace{dx}_{\text{change in } x}$$

Let $u = 1 + x^2$

$\rightsquigarrow du = 2x\,dx$

$\rightsquigarrow dx = \dfrac{du}{2x}$

Substitute $$\int x\sqrt{1+x^2}\,dx = \int x\sqrt{u}\,\frac{du}{2x} = \frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\,\frac{u^{3/2}}{3/2} + C$$

$$= \frac{1}{3} u^{3/2} + C = \boxed{\frac{1}{3}(1+x^2)^{3/2} + C}$$

Double check.

$$\left[\frac{1}{3}(1+x^2)^{3/2}\right]' = \frac{1}{3} 2x \frac{3}{2} (1+x^2)^{1/2} = x(1+x^2)^{1/2} = x\sqrt{1+x^2}$$

Summarize

Goal: find $\int f(x)\,dx$

Step 1. introduce a new variable $u = u(x)$

Step 2. compute the differential $du = u'(x)\,dx$

$$\leadsto dx = \frac{du}{u'(x)}$$

Step 3 substitute u and dx into $\int f(x)\,dx$

[u-substitution fails if x is still found in the integral.]

Step 4 evaluate the integral

Step 5 substitute back $u = u(x)$

Ex

$$\int \frac{2x+1}{\sqrt{x^2+x+1}}\,dx$$

$u = x^2+x+1$

$du = (2x+1)\,dx$

$$\int \frac{\overbrace{2x+1 \; dx}^{du}}{\sqrt{x^2+x+1}} = \int \frac{du}{\sqrt{u}} = \int u^{-1/2}\,du = \frac{u^{1/2}}{1/2} + C = 2\sqrt{x^2+x+1} + C$$

Note u-substitution can solve a lot of complicated problem, but not all

There is no general rule as to how to select a substitution. The more we practice, the more experienced we are.