

Lecture 12

Monday, January 30, 2023 8:20 AM

* Questions ----

Substitution rule practice: (See worksheet 1)

$$\underline{\underline{\int x \sqrt{1-x} dx}}$$

Let $u = 1-x$. Then

$$du = u'dx = -dx$$

$$\begin{aligned} \int x \sqrt{1-x} dx &= \int x \sqrt{u} (-du) = \int (1-u) \sqrt{u} (-du) = \int (u-1) \sqrt{u} du \\ &= \int (u \sqrt{u} - \sqrt{u}) du \\ &= \int (u^{3/2} - u^{1/2}) du = \frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2} + C \\ &= \frac{(1-x)^{5/2}}{5/2} - \frac{(1-x)^{3/2}}{3/2} + C \end{aligned}$$

* Substitution rule for definite integral $\int_a^b f(x) dx$

$$\underline{\underline{\int_{-1}^0 \sqrt{3x+4} dx}}$$

$$u = 3x+4$$

$$du = u'dx = 3dx$$

$$dx = \frac{1}{3} du$$

x	-1	0
u	1	4

Substitute:

$$\begin{aligned} \int_{-1}^0 \sqrt{3x+4} dx &= \int_1^4 \sqrt{u} \frac{1}{3} du = \frac{1}{3} \frac{u^{3/2}}{3/2} \Big|_1^4 \\ &= \frac{1}{3} \left(\frac{4^{3/2}}{3/2} - \frac{1^{3/2}}{3/2} \right) = \frac{14}{9} \end{aligned}$$

Procedure to find $\int_a^b f(x) dx$ using the substitution rule.

Step 1 Introduce a new variable $u = u(x)$

Step 2 $du = u' dx$

$$dx = \frac{du}{u'}$$

Step 3 Update the integral bounds

x	a	b
u	$?$	$?$

Step 4 Substitute u and dx into the integral

$$\int_a^b f(x) dx = \int_{?}^{?} \dots du$$

Step 5 Evaluate the integral

This procedure is somewhat like an instruction of tax return...

Ex

$$\int_0^1 \sqrt[3]{1+7x} dx$$

$$u = 1+7x$$

$$du = 7 dx$$

$$dx = \frac{1}{7} du$$

x	0	1
u	1	8

Substitute

$$\int_0^1 \sqrt[3]{1+7x} dx = \int_1^8 \sqrt[3]{u} \frac{1}{7} du = \frac{3}{4} \frac{1}{7} u^{\frac{4}{3}} \Big|_1^8 = \frac{45}{28}$$