

Lecture 13

Tuesday, January 31, 2023 8:25 AM

* Questions ...

We have learned how to integrate a function.

Overview:

$\int f(x) dx$: antiderivative of f , or indefinite integral of f

$\int_a^b f(x) dx$: definite integral of f

How to evaluate them? we have learned several methods.

Ex

$$\int_0^{\pi/2} \sin x \cos x dx$$

$$u = \sin x$$

$$du = u' dx = \cos x dx$$

$$dx = \frac{du}{\cos x}$$

x	0	$\pi/2$
u	0	1

$$\begin{aligned} \int_0^{\pi/2} \sin x \cos x dx &= \int_0^1 u \cos x \frac{du}{\cos x} = \int_0^1 u du \\ &= \frac{u^2}{2} \Big|_0^1 = \frac{1}{2} \end{aligned}$$

Ex

$$\int_0^1 (x+1) \sqrt{x^2+2x} dx$$

$$u = x^2+2x$$

$$du = u' dx = (2x+2) dx$$

$$dx = \frac{du}{2x+2}$$

x	0	1
u	0	3

$$\begin{aligned} \int_0^1 (x+1) \sqrt{x^2+2x} dx &= \int_0^3 (x+1) \sqrt{u} \frac{du}{2x+2} \\ &= \int_0^3 (x+1) \sqrt{u} \frac{du}{2(x+1)} \\ &= \int_0^3 \frac{\sqrt{u}}{2} du = \frac{1}{2} \frac{u^{3/2}}{3/2} \Big|_0^3 = \sqrt{3} \end{aligned}$$

Inverse function

$$\left. \begin{array}{l} f(x) = y \\ g(y) = x \end{array} \right\} f \text{ and } g \text{ are inverses of each other.}$$

$$f(g(x)) = x$$

$$g(f(x)) = x$$