Lecture 19
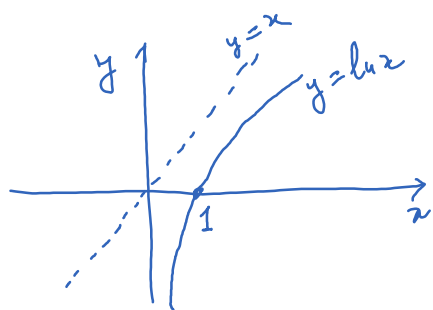
* Questions...

* The challenge problem

$$f(x) = x - \ln x, \quad x > 0$$

$$f'(x) = 1 - \frac{1}{x} = \frac{x-1}{x}$$

$$f(x) \geq 1 \text{ for all } x > 0$$

Thus, the two graphs have no intersection with each other.

Exponential functions:

$$e^x \longrightarrow \text{inverse of } \ln x$$

$$(e^x)' = e^x, \quad (e^u)' = u' e^u$$

$$(e^{2x})' = 2e^{2x}$$

$$(e^{ax})' = a e^{ax}$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$$

General exponential function:

$$a^x = \left(e^{\ln a}\right)^x = e^{x\ln a}.$$

$$\left(a^x\right)' = \left(e^{x\ln a}\right)' = (\ln a)\, e^{x\ln a} = a^x \ln a$$

$$\int a^x\,dx = \frac{1}{\ln a}\, a^x + C$$

* Food for thought:

what is $\left(x^x\right)'$?