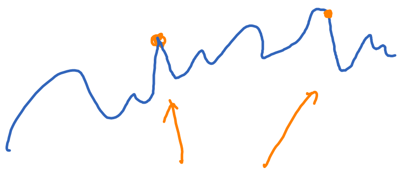
Lecture 2

Tuesday, January 10, 2023 8:18 AM

$f(x) \rightsquigarrow f'(x)$: instantaneous rate of change



not differentiable

For a function to be differentiable everywhere, the graph has to be "smooth" enough.

The reverse of derivative is called antiderivative.

Given function $f(x)$, the function $g(x)$ satisfying $g'(x) = f(x)$ is called an <u>antiderivative</u> of $f(x)$.

How do we find antiderivative?

<u>Ex</u> $f(x) = x^2 + x + 1$

$$(x^3)' = 3x^2 \rightsquigarrow \frac{1}{3}(x^3)' = x^2 \rightsquigarrow \left(\frac{1}{3}x^3\right)' = x^2$$

$$(x^2)' = 2x \rightsquigarrow \frac{1}{2}(x^2)' = x \rightsquigarrow \left(\frac{1}{2}x^2\right)' = x$$

$$(x)' = 1$$

Therefore, $$\Big(\underbrace{\frac{1}{3}x^3 + \frac{1}{2}x^2 + x}_{\text{antiderivative of } f}\Big)' = x^2 + x + 1 = f(x)$$

Note that antiderivatives are not unique.

Any function of the form $\underbrace{\frac{x^3}{3} + \frac{x^2}{2} + x}_{g(x)} + C$, where C is a constant is an antiderivative.

Are there other antiderivatives that are not of the above form?

$$h'(x) = f(x) = g'(x)$$

$$\rightsquigarrow \quad [h(x) - g(x)]' = 0$$

$$\rightsquigarrow \quad h(x) - g(x) = \text{const}$$

$$\rightsquigarrow \quad h(x) = g(x) + \text{const}$$

Ex (a) Find all antiderivatives of $f(x) = x^{3/2} - \sqrt{x}$.

Answer: $\frac{2}{5} x^{5/2} - \frac{2}{3} x^{3/2} + C$

(b) Find an antiderivative $F(x)$ such that $F(1) = 2$

$$F(1) = \frac{2}{5} - \frac{2}{3} + C = 2 \quad \rightsquigarrow \quad C = \frac{34}{15}$$

$$\rightsquigarrow \quad F(x) = \frac{2}{5} x^{5/2} - \frac{2}{3} x^{3/2} + \frac{34}{15}$$

Useful antiderivatives:

$$\int x^\alpha \, dx = \frac{x^{\alpha+1}}{\alpha+1} + C \qquad (\alpha \neq -1)$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$