Lecture 20

Thursday, February 16, 2023 3:15 AM

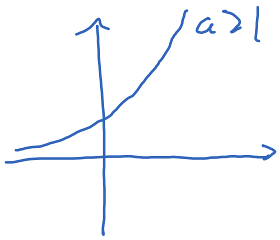
* Questions ....

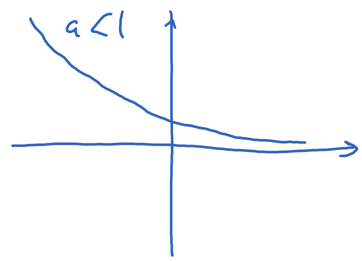
$$x^x = \left(e^{\ln x}\right)^x = e^{x\ln x}$$

$$(x^x)' = \left(e^{x\ln x}\right)' = (x\ln x)' e^{x\ln x} = \left(\ln x + x\frac{1}{x}\right) e^{x\ln x}$$

$$= (1+\ln x)\, x^x$$

* a^x is an increasing function if $a > 1$, decreasing if $0 < a < 1$.





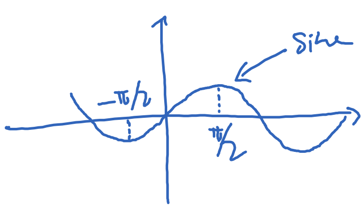
* Inverse trigonometric functions:

$\sin x \longrightarrow \arcsin x,\ \sin^{-1} x$

$\cos x \longrightarrow \arccos x,\ \cos^{-1} x$

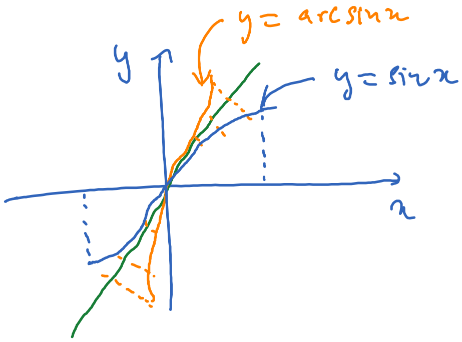
$\tan x \longrightarrow \arctan x,\ \tan^{-1} x$

$\cot x \longrightarrow \operatorname{arccot} x,\ \cot^{-1} x$



Note that the sine function doesn't pass the Horizontal Line Test. We need to restrict the domain of sine.

Sine function passes the horizontal line test if $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.



The domain of arcsin is $[-1, 1]$.

The range of arcsin is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

How about its derivative?

$$\sin(\arcsin x) = x$$

$$\rightsquigarrow \quad [\sin(\arcsin x)]' = x'$$

$$\rightsquigarrow \quad \underbrace{\cos(\arcsin x)}_{=\sqrt{1-x^2}} (\arcsin x)' = 1$$

$$\rightsquigarrow \quad (\arcsin x)' = \frac{1}{\sqrt{1-x^2}}.$$

<u>Derivative of $f(x)^{g(x)}$</u>

$f(x)^{g(x)}$ is neither an exponential function or a power function.

- Write $f(x)^{g(x)} = \left[e^{\ln f(x)}\right]^{g(x)} = e^{g(x)\ln f(x)}$
- $\left[e^{g(x)\ln f(x)}\right]' = [g(x)\ln f(x)]' \underbrace{e^{g(x)\ln f(x)}}_{f(x)^{g(x)}}$

Ex $\left[(\sin x)^{\cos x}\right]' = ?$

$\left(x^{x^2}\right)' = ?$