Lecture 21

Friday, February 17, 2023 8:38 AM

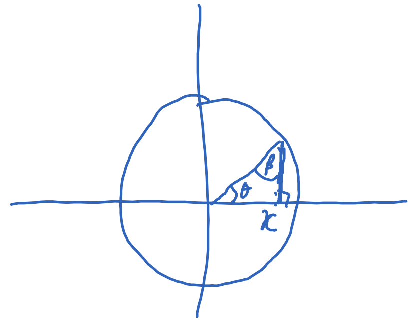
* Questions.

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$(\operatorname{arccot} x)' = \frac{-1}{1+x^2}$$



$\theta = \arcsin x$

$\beta = \arccos x$

$\theta + \beta = \frac{\pi}{2}$

$\rightsquigarrow \arcsin x + \arccos x = \frac{\pi}{2}$

$\rightsquigarrow (\arcsin x)' + (\arccos x)' = 0$

Ex

$$\int_{-1}^{0} \frac{2}{x^2+2x+2}\,dx = \int_{-1}^{0} \frac{2}{(x+1)^2+1}\,dx$$

Let $u = x+1$.

$du = u'dx = dx$

x	-1	0
u	0	1

$$= \int_{0}^{1} \frac{2}{u^2+1}\,du$$

$$= 2\arctan u \Big|_0^1$$

$$= 2(\arctan 1 - \arctan 0)$$

$$= 2\left(\frac{\pi}{4} - 0\right)$$

$$= \frac{\pi}{2}.$$

Ex

$$\int_{-1}^{1} \frac{2}{x^2+2x+5}\,dx = \int_{-1}^{1} \frac{2}{(x+1)^2+4}\,dx$$

$u = x+1$

$du = dx$

x	-1	1
u	0	2

$$= \int_{0}^{2} \frac{2}{u^2+4}\,du$$

$$= \int_{0}^{2} \frac{2/4}{\frac{u^2}{4}+1}\,du$$

$$= \frac{1}{2}\int_{0}^{2} \frac{du}{\left(\frac{u}{2}\right)^2+1}$$

Let $v = \frac{u}{2}$

$dv = \frac{du}{2}$

$du = 2dv$

u	0	2
v	0	1

$$= \frac{1}{2}\int_{0}^{1} \frac{2\,dv}{v^2+1}$$

$$= \int_{0}^{1} \frac{dv}{v^2+1}$$

$$= \arctan v \Big|_{0}^{1} = \frac{\pi}{4}.$$

L'Hopital rule

This is a rule that helps us find limits in an indeterminate form $\frac{0}{0}, \frac{\infty}{\infty}, \frac{-\infty}{\infty}, \frac{\infty}{-\infty}$.

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$