Lecture 24

Thursday, February 23, 2023 1:47 AM

* Questions ....

More practice with L'Hospital rule:

Ex $\lim_{x\to 0^+} \sin x \ln x = \underbrace{\lim_{x\to 0^+} \frac{\sin x}{x}}_{=1} \lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} x\ln x$

$= \lim_{x\to 0^+} \frac{\ln x}{1/x} \overset{L'H}{=} \lim_{x\to 0^+} \frac{1/x}{-1/x^2} = \lim_{x\to 0^+} (-x) = 0.$

Ex $\lim_{x\to\infty} \left[x - x^2 \ln\left(\frac{1+x}{x}\right)\right]$

Let $t = \frac{1}{x} \to 0$ as $x\to\infty$.

$$x - x^2 \ln\left(\frac{1+x}{x}\right) = \frac{1}{t} - \frac{1}{t^2}\ln(t+1) = \frac{t - \ln(t+1)}{t^2}$$

$$\lim_{t\to 0} \frac{t-\ln(t+1)}{t^2} \overset{L'H}{=} \lim_{t\to 0} \frac{1 - \frac{1}{t+1}}{2t} = \lim_{t\to 0} \frac{1}{2(t+1)} = \frac{1}{2}.$$

Integration by parts

$$(uv)' = u'v + uv'$$

$$\int (uv)' dx = \int u'v\,dx + \int uv'$$

$$\rightsquigarrow \boxed{\int u'v\,dx = uv - \int uv'\,dx}$$

In an equivalent form,

$$\boxed{\int v\,du = uv - \int u\,dv}$$

Ex $\int xe^x dx$

Ex $\int \ln x\, dx$

Ex $\int x\cos x\, dx$

Ex $\int x\ln x\, dx$