Lecture 25

Friday, February 24, 2023 10:34 AM

* Questions

Find integrals $\begin{cases} \int f(x)\,dx \\ \int_a^b f(x)\,dx \end{cases}$

Method 1: use the table

Method 2. substitution

Method 3: integration by parts ← we will learn this today.

The method of integration by parts stems from the product rule of differentiation

$$(uv)' = u'v + uv'$$

Integrate both sides:

$$\int (uv)'\,dx = \int u'v\,dx + \int uv'\,dx$$

$$uv = \int u'v\,dx + \int uv'\,dx$$

$$\boxed{\underbrace{\int u'v\,dx}_{\text{the problem to deal with}} = uv - \underbrace{\int uv'\,dx}_{\text{a new problem}}}$$ ← Integration by part

Ex $\int \underbrace{xe^x}_{u'v}\,dx$ We can either choose $\begin{cases} u' = x \\ v = e^x \end{cases}$ or $\begin{cases} u' = e^x \\ v = x \end{cases}$

One way is easier than the other.

If we chose $u' = x$ and $v = e^x$ then $u = \frac{x^2}{2}$ and $v' = e^x$.

$$\int x e^x dx = \int u' v dx = uv - \int u v' dx = \frac{x^2}{2} e^x - \underbrace{\int \frac{x^2}{2} e^x dx}$$

a more difficult problem to deal with

If we choose $u' = e^x$ and $v = x$ then $u = e^x$ and $v' = 1$.

$$\int x e^x = x e^x - \int e^x dx = x e^x - e^x + C$$

Ex

$$\int x \ln x \, dx$$

$u' = x \longrightarrow u = \frac{x^2}{2}$

$v = \ln x \longrightarrow v' = \frac{1}{x}$

$$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \frac{1}{x} dx = \frac{x^2}{2} \ln x - \int \frac{x}{2} dx$$

$$= \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$

Ex

$$\int \ln x \, dx$$

$u' = 1 \longrightarrow u = x$

$v = \ln x \longrightarrow v' = \frac{1}{x}$

$$\int \ln x \, dx = x \ln x - \int x \frac{1}{x} dx = x \ln x - \int 1 \, dx = x \ln x - x + C$$

Ex

$$\int \arcsin x \, dx$$

$u' = 1 \longrightarrow u = x$

$v = \arcsin x \longrightarrow v' = \frac{1}{\sqrt{1-x^2}}$

$$\int \arcsin x \, dx = x \arcsin x - \underbrace{\int \frac{x}{\sqrt{1-x^2}}\, dx}_{\text{use substitution } u = 1-x^2}$$

$$du = u'\, dx = -2x\, dx$$

$$dx = \frac{du}{-2x}$$

$$\int \frac{x}{\sqrt{1-x^2}}\, dx = \int \frac{x}{\sqrt{u}} \frac{du}{-2x} = -\frac{1}{2} \int \frac{1}{\sqrt{u}}\, du = -\frac{1}{2} \int u^{-1/2}\, du$$

$$= -\frac{1}{2} \frac{u^{1/2}}{1/2} + C$$

$$= -u^{1/2} + C$$

$$= -\sqrt{1-x^2} + C$$

Therefore,

$$\int \arcsin x\, dx = x \arcsin x + \sqrt{1-x^2} + C$$

Next time, we'll learn how to use the integration by parts to find definite integrals.