

Lecture 28

Thursday, March 2, 2023 4:02 PM

* Questions.

Trigonometric substitution:

$$\underline{\text{Ex}} \quad \int_0^1 \sqrt{1-x^2} dx$$

$$\text{Let } x = \sin u$$

$$dx = x' du = \cos u du$$

x	0	1
u	0	$\frac{\pi}{2}$

$$\int_0^1 \sqrt{1-x^2} dx = \int_0^{\pi/2} \sqrt{1-\sin^2 u} \cos u du = \int_0^{\pi/2} \cos^2 u du$$

$$= \int_0^{\pi/2} \frac{1+\cos 2u}{2} du$$

$$= \left(\frac{u}{2} + \frac{\sin 2u}{4} \right) \Big|_0^{\pi/2} = \frac{\pi}{4}$$

$$\underline{\text{Ex}} \quad \int_{\pi/2}^{\pi/4} \frac{\sin x}{1-\cos x} dx = \int_1^{1-\frac{\sqrt{2}}{2}} \frac{du}{u} = \ln u \Big|_1^{1-\frac{\sqrt{2}}{2}} = \ln\left(1-\frac{\sqrt{2}}{2}\right).$$

$$u = 1 - \cos x$$

$$du = \sin x dx$$

x	$\pi/2$	$\pi/4$
u	1	$1-\frac{\sqrt{2}}{2}$

* Integral of rational functions.

A rational function is a quotient of two polynomials: $f(x) = \frac{P(x)}{Q(x)}$.

Ex.

Any constant C is a rational function

Any polynomial is a rational function

$$\frac{1}{x}, \frac{x^2+1}{x^3}, \frac{-1+x^3}{x^2+2}, \dots$$

To integrate a rational function, you may need the following skills:

- * Factoring a polynomial
- * Long division, synthetic division
- * Partial fractional decomposition

Review these topics by watching the videos . .