Lecture 29

Thursday, March 2, 2023 10:56 PM

* Questions ...

* Long division:

$$\frac{P(x)}{Q(x)} = \underbrace{q(x)}_{\text{quotient}} + \frac{R(x)}{Q(x)} \leftarrow \text{remainder}$$

To find $q(x)$ and $R(x)$, we use long division. If $Q(x)$ is of degree 1, we can use synthetic division (simpler than long division).

$$\frac{x^4 - 2x^3 + x - 1}{x^2 + x - 1}$$

$$
\begin{array}{r|l}
 & x^2 - 3x + 4 \quad \leftarrow \text{quotient} \\
\hline
x^2 + x - 1 & x^4 - 2x^3 + 0x^2 + x - 1 \\
 & -\;(x^4 + x^3 - x^2) \\
\hline
 & -3x^3 + x^2 + x \\
 & -\;(-3x^3 - 3x^2 + 3x) \\
\hline
 & 4x^2 - 2x - 1 \\
 & -\;(4x^2 + 4x - 4) \\
\hline
 & -6x + 3 \quad \leftarrow \text{remainder}
\end{array}
$$

$$\frac{x^4-2x^3+x-7}{x^2+x-1} = x^2-3x+4 + \frac{-6x+3}{x^2+x-1}$$

Ex

$$\frac{x^3+x+1}{x-2} = x^2+2x+5+\frac{11}{x-2}$$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 1 & 1 \\ & & 2 & 4 & 10 \\ \hline & \underbrace{1 \quad 2 \quad 5}_{\text{quotient}} & & & \underbrace{11}_{\text{remainder}} \end{array}$$

$$\int \frac{x^3+x+1}{x-2}\,dx = \int \left(x^2+2x+5+\frac{11}{x-2}\right)dx$$

$$= \frac{x^3}{3}+x^2+5x+11\ln|x-2|+C$$

Recall the goal find $\int \frac{P(x)}{Q(x)}\,dx$.

Rules. ① Simplify the fraction first. You will need to factor $Q(x)$.

② If the degree of $P(x)$ is less than the degree of $Q(x)$ then we need to do partial fraction decomposition

Ex $\int \frac{x}{x^2+x-2}\,dx = ?$

$$\frac{x}{x^2+x-2} = \frac{x}{(x-1)(x+2)} = \underbrace{\frac{A}{x-1} + \frac{B}{x+2}}_{\text{common denominator is } (x-1)(x+2)}, \text{ where } A, B \text{ are to be determined.}$$

$$\frac{x}{(x-1)(x+2)} = \frac{A(x+2)}{(x-1)(x+2)} + \frac{B(x-1)}{(x-1)(x+2)}$$

$$\longrightarrow \quad x = A(x+2) + B(x-1)$$

Plug $x=1$: $\quad 1 = A(3) \longrightarrow A = \frac{1}{3}$

Plug $x=-2$: $\quad -2 = B(-3) \longrightarrow B = \frac{2}{3}$

Thus,

$$\underbrace{\frac{x}{(x-1)(x+2)} = \frac{1/3}{x-1} + \frac{2/3}{x+2}}_{\text{This is partial fraction decomposition}}$$

$$\int \frac{x}{(x-1)(x+2)}\,dx = \int \left(\frac{1/3}{x-1} + \frac{2/3}{x+2}\right)dx$$

$$= \frac{1}{3}\ln|x-1| + \frac{2}{3}\ln|x+2| + C$$

Recall:

① $\displaystyle\int \frac{1}{x+a}\,dx = \ln|x+a| + C$

② $\displaystyle\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + C$

Ex $\displaystyle\int \frac{2}{x^2+4x+3}\,dx = ?$

$$\frac{2}{x^2+4x+3} = \frac{2}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}$$

$$\rightsquigarrow \quad 2 = A(x+3) + B(x+1)$$

Plug $x=-1$. $\quad 2 = A(2) \longrightarrow A = 1$

Plug $x=-3$. $\quad 2 = B(-2) \longrightarrow B = -1$

$$\int \frac{2}{x^2+4x+3}\,dx = \int\left(\frac{1}{x+1} - \frac{1}{x+3}\right)dx = \ln|x+1| - \ln|x+3| + C$$

Ex

$$\int \frac{x}{x^2+2x+1}\,dx = ?$$

$$\frac{x}{x^2+2x+1} = \frac{x}{(x+1)^2} = \cancel{\frac{A}{x+1} + \frac{B}{x+1}}$$

$$= \underbrace{\frac{A}{x+1} + \frac{B}{(x+1)^2}}_{\text{Common denominator is } (x+1)^2}$$

$$\frac{x}{(x+1)^2} = \frac{A(x+1)+B}{(x+1)^2}$$

$$\longrightarrow \quad x = A(x+1) + B$$

Plug $x=-1$. $\quad -1 = B$

Plug $x=0$. $\quad 0 = A+B = A-1 \longrightarrow A = 1$

Therefore,

$$\int \frac{x}{(x+1)^2}\,dx = \int \left(\frac{1}{x+1} - \frac{1}{(x+1)^2}\right) dx$$

$$= \ln|x+1| + \frac{1}{x+1} + C$$