Lecture 3

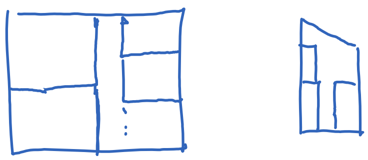
Wednesday, January 11, 2023 6:33 PM

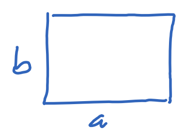
Question .....

Last time: antiderivative

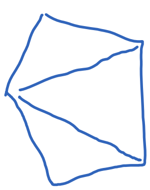
can be used to compute length, area, volume of different shapes.

Calculate the square footage of a house: you divide the floor plan into rectangles or triangles.



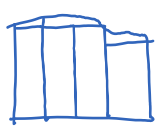


area = ab

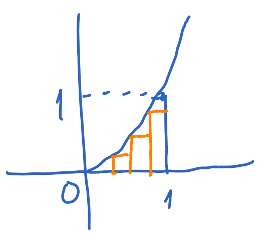


For complicated shape, you need to approximate.

This is the idea of Archimedes (200s BC),



flourished by Riemann (1850s).

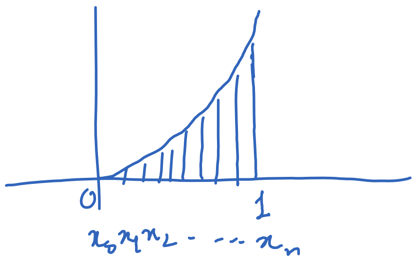


$$L_4 = 0 \times 0 + 0.25 \times 0.25^2 + 0.5 \times 0.5^2 + 0.75 \times 0.75^2$$

$$\approx .....$$

$$R_4 = 0.25 \times 0.25^2 + 0.5 \times 0.5^2 + 0.75 \times 0.75^2 + 1 \times 1^2$$

$$\approx ...$$



$x_0 = 0$

$x_1 = \frac{1}{n}$

$x_2 = \frac{2}{n}$

$\cdots$

$x_n = \frac{n}{n} = 1$

$$x_k = \frac{k}{n}$$

$$\text{area} \approx L_n = \frac{1}{n} \times x_0^2 + \frac{1}{n} \times x_1^2 + \cdots + \frac{1}{n} \times x_{n-1}^2$$

$$= \frac{1}{n}\left(x_0^2 + x_1^2 + \cdots + x_{n-1}^2\right)$$

$$= \frac{1}{n} \sum_{k=0}^{n-1} x_k^2 = \frac{1}{n} \sum_{k=0}^{n-1} \left(\frac{k}{n}\right)^2$$

In general,

$$\underbrace{L_n = \sum_{k=0}^{n-1} f(x_k) \Delta x}_{\text{left-point Riemann sum}}, \qquad \underbrace{R_n = \sum_{k=1}^{n} f(x_k) \Delta x = \sum_{k=0}^{n-1} f(x_{k+1}) \Delta x}_{\text{right-point Riemann sum}}$$

Mathematica:

```
n = 10;
Sum[ 1/n (k/n)^2, {k, 0, n-1}]
```