Lecture 30

Monday, March 6, 2023 6:27 AM

* Questions ....

* Integration of rational function

$$\int \frac{P(x)}{Q(x)}\,dx$$

If $Q(x)$ can be factored as $(x-a_1)(x-a_2)\ldots(x-a_n)$, where $a_1, a_2, \ldots, a_n$ are distinct, and if the degree of P is less than the degree of Q then one will do partial fractional decomposition:

$$\frac{P(x)}{Q(x)} = \frac{A_1}{x-a_1} + \frac{A_2}{x-a_2} + \cdots + \frac{A_n}{x-a_n}$$

where $A_1, A_2, \ldots, A_n$ are constants to be determined.

Ex:

$$\frac{x^2+1}{x(x-1)(x-2)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-2}$$

Ex

$$\frac{x^2+1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

Ex

$$\frac{3x+4}{x^2(x-1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$$

Ex

$$\frac{2x-1}{x^2+1} = \frac{2x}{x^2+1} - \frac{1}{x^2+1}$$

$$\int \frac{2x-1}{x^2+1}\,dx = \underbrace{\int \frac{2x}{x^2+1}\,dx}_{\text{let } u = x^2+1} - \underbrace{\int \frac{1}{x^2+1}\,dx}_{\arctan x}$$

$$= \int \frac{du}{u} - \arctan x$$

$$= \ln|u| - \arctan x + C$$

$$= \ln(x^2+1) - \arctan x + C$$

Ex

$$\frac{x}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

Ex

$$\frac{2x+1}{x(x-1)(x^2+2x+2)} = \frac{A}{x} + \frac{B}{x-1} + \frac{Cx+D}{x^2+2x+2}$$