Lecture 31

Tuesday, March 7, 2023 8:14 AM

* Questions....

$$\int \frac{P(x)}{Q(x)}\,dx = ?$$

Sometimes, the denominator can't be factored as $(x-a_1)(x-a_2)\cdots(x-a_n)$.

For example,

$$\int_0^1 \frac{x}{x^2-2x+5}\,dx$$

In this case, we complete the square in the denominator:

$$x^2-2x+5 = (x-1)^2+4$$

And then use substitutions.

$$\int_0^1 \frac{x}{(x-1)^2+4}\,dx \overset{u=x-1}{=\!=} \int_{-1}^{0} \frac{u+1}{u^2+4}\,du$$

Then split this into two integrals:

$$\int_{-1}^{0} \frac{u+1}{u^2+4}\,du = \underbrace{\int_{-1}^{0} \frac{u}{u^2+4}\,du}_{\text{use substitution } v=u^2+4} + \underbrace{\int_{-1}^{0} \frac{1}{u^2+4}\,du}_{\text{use substitution } v=\frac{u}{2}}$$

Ex $$\int \frac{x}{x^3+x^2-2}\,dx = \int \frac{x}{(x-1)(x^2+2x+2)}\,dx$$

$$\frac{x}{(x-1)(x^2+2x+2)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+2x+2}$$