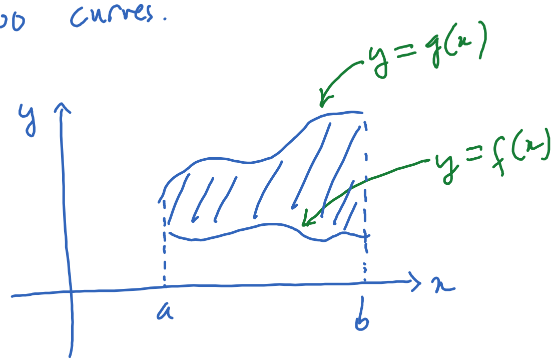
Lecture 35

Monday, March 13, 2023 3:53 PM

* Questions...

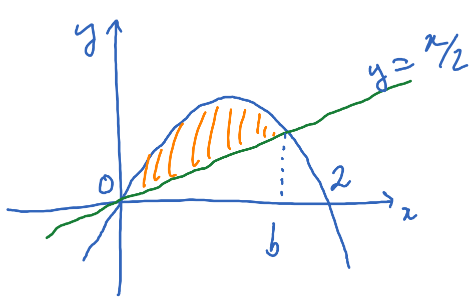
Integral can be used to find the area of the region between two curves.



$$\text{Area} = \int_a^b g(x)\,dx - \int_a^b f(x)\,dx$$

$$= \int_a^b (g(x) - f(x))\,dx$$

__Ex__ Find the area bounded by the parabola $y = 2x - x^2$ and the line $y = \frac{x}{2}$.



To find the intersection of the parabola and the line, we solve

$$2x - x^2 = \frac{x}{2}$$

$$\rightsquigarrow \frac{3}{2}x - x^2 = 0$$

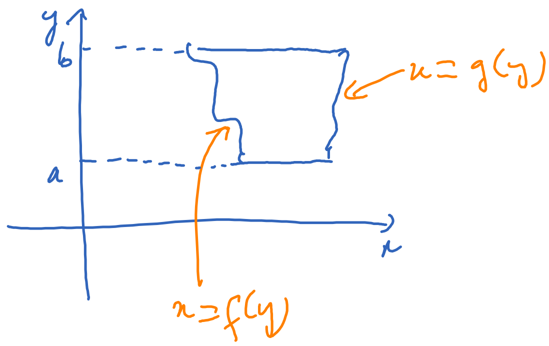
$$\rightsquigarrow x = 0 \quad \text{or} \quad x = \frac{3}{2}$$

$$\rightsquigarrow b = \frac{3}{2}$$

$$\text{Area} = \int_a^b (\text{upper} - \text{lower})\,dx = \int_0^{3/2} \left(2x - x^2 - \frac{x}{2}\right) dx$$

$$= \int_0^{3/2} \left(\frac{3x}{2} - x^2\right) dx = \left(\frac{3x^2}{4} - \frac{x^3}{3}\right)\Big|_0^{3/2} = \frac{9}{16}.$$

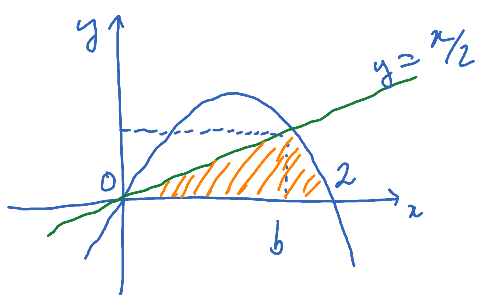
Sometimes, a region can be described as the region between two curves $x = g(y)$ and $x = h(y)$, where $a \le y \le b$.



$$\text{area} = \int_a^b \big(g(y) - f(y)\big)\,dy$$

If you turn the paper 90°, you will see that we get back the above region.

<u>Ex</u>



The lower curve is $y = 0$.
The upper curve is sometimes $y = \frac{x}{2}$, sometimes $y = 2x - x^2$.

In other to know which upper curve to use, we can split the region into two sub-region. Alternatively, you can re-describe the region:

$$2y \le x \le \ ?$$
$$0 \le y \le \tfrac{3}{2}$$

To find the upper bound for x, we need to solve for x from the equation $y = 2x - x^2$

$\rightarrow \quad x^2 - 2x + y = 0$

$\rightarrow \quad x = 1 \pm \sqrt{1-y}$, pick the "+" sign

$$\text{Area} = \int_0^{3/4} \Big(1 + \underbrace{\sqrt{1-y}}_{\substack{\text{upper} \\ \text{curve}}} - \underbrace{2y}_{\substack{\text{lower} \\ \text{curve}}} \Big)\, dy = \cdots$$