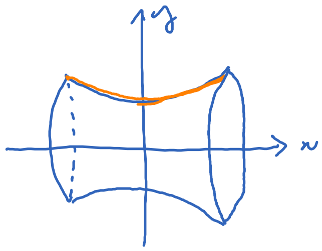
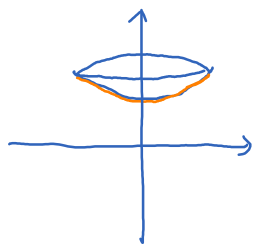
Lecture 36

Thursday, March 16, 2023 3:53 PM

* Questions...

Integral can be used to find the volume of a solid of revolution.

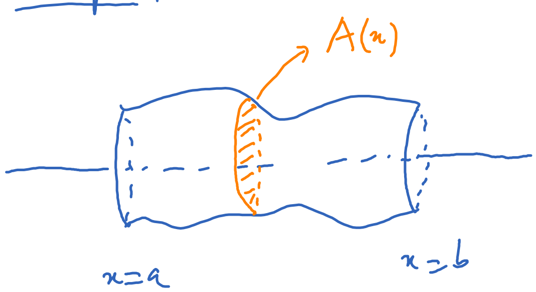




The orange curve is rotated about the x-axis

Rotation about the y-axis

Principle:

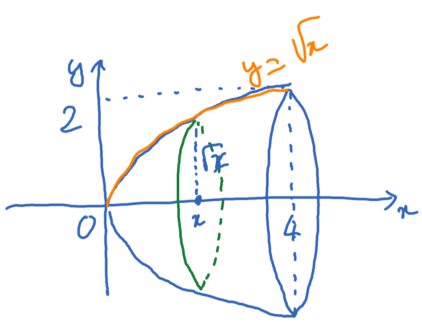


Consider a solid that stretches from $x=a$ to $x=b$ on the x-axis.

Let $A(x)$ be the area of the cross section at position x. Then the volume of the solid is

$$\text{vol} = \int_a^b A(x)\,dx.$$

Ex:



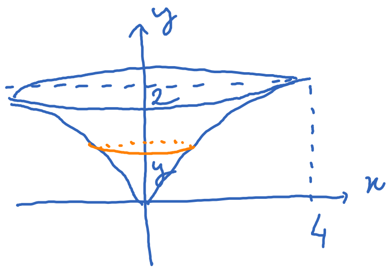
Each cross section is a disk.

At position x, the cross section has radius $r = \sqrt{x}$.

$$A(x) = \pi r^2 = \pi \sqrt{x}^2 = \pi x.$$

$$\text{vol} = \int_0^4 A(x)\,dx = \int_0^4 \pi x\,dx = \frac{\pi x^2}{2}\Big|_0^4 = 8\pi$$

Ex

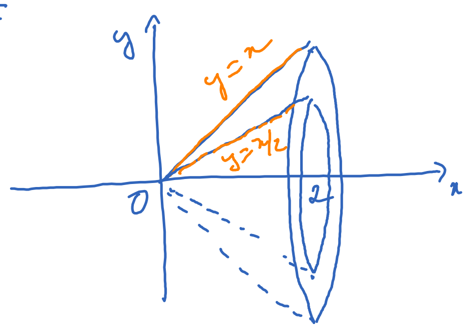


The same curve $y=\sqrt{x}$ is now rotated about the y-axis. Each cross section is a disk. The cross section at position y has radius $r = x = y^2$.

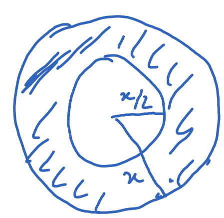
$$A(y) = \pi r^2 = \pi (y^2)^2 = \pi y^4.$$

$$\text{vol} = \int_0^2 A(y)\,dy = \int_0^2 \pi y^4\,dy = \frac{32\pi}{5}.$$

Ex



Each cross section is an annulus. At position x, the inner radius is $\frac{x}{2}$ and the outer radius is x.



$$A(x) = \pi x^2 - \pi\left(\frac{x}{2}\right)^2 = \frac{3\pi}{4}x^2$$

$$\text{vol} = \int_0^2 A(x)\,dx = \int_0^2 \frac{3\pi x^2}{4}\,dx = \frac{\pi x^3}{4}\Big|_0^2 = 2\pi$$