Lecture 5

Friday, January 13, 2023 8:14 AM

* Question

Introduction to Mathematica..

Use it either on Jupyter notebook or WolframCloud.com

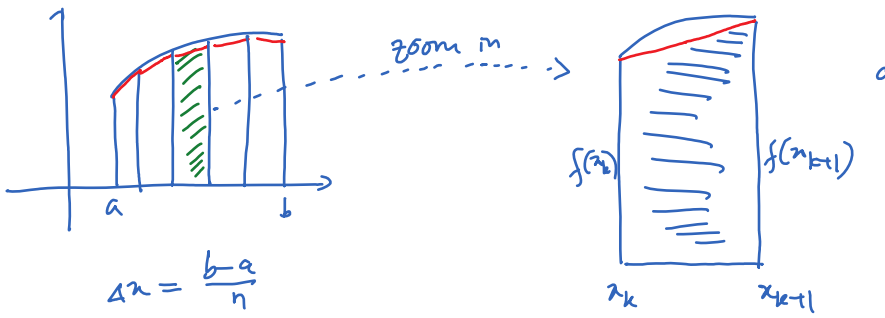
* Riemann sums

Left-point $L_n = \sum_{k=0}^{n-1} f(x_k)\Delta x$

Right-point. $R_n = \sum_{k=1}^{n} f(x_k)\Delta x$

Mid-point $M_n = \sum_{k=0}^{n-1} f\left(\frac{x_k + x_{k+1}}{2}\right)\Delta x$

Trapezoid. $T_n = \sum_{k=0}^{n-1} \frac{f(x_k)+f(x_{k+1})}{2}\Delta x$



area of slat $\approx$ area of trapezoid

$= \frac{1}{2}\left(f(x_k)+f(x_{k+1})\right)\Delta x$

The exact area of the region under the curve is $\lim_{n\to\infty} L_n$, or $\lim_{n\to\infty} R_n$, or $\lim_{n\to\infty} M_n$, or $\lim_{n\to\infty} T_n$.

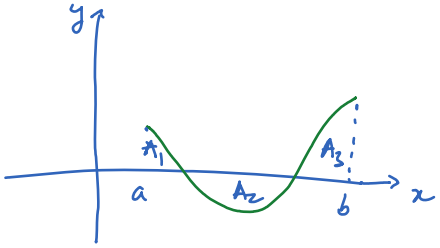
Notation $\int_a^b f(x)dx = \lim_{n\to\infty} L_n = \lim_{n\to\infty} R_n = \lim_{n\to\infty} M_n = \lim_{n\to\infty} T_n$

This is called the integral of f from a to b.

integral bounds: $\int_a^b f(x)\,dx$ (definite integral)

$f(x)$: integrand

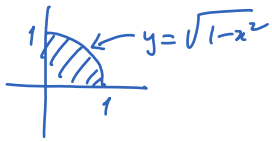
This is the algebraic area under graph of f and above the interval $[a,b]$.



$$\int_a^b f(x)\,dx = A_1 - A_2 + A_3$$

Ex $\int_0^1 \sqrt{1-x^2}\,dx$

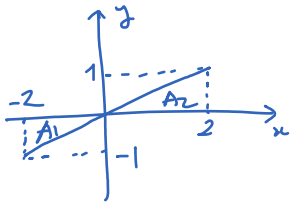
$y = \sqrt{1-x^2} \rightsquigarrow y^2 = 1-x^2 \rightsquigarrow x^2+y^2 = 1$



$= \frac{1}{4}$ area of circle

$= \frac{1}{4}\pi = \frac{\pi}{4}$

Ex



$$\int_{-2}^{2} \frac{x}{2}\,dx = A_2 - A_1 = 0$$

$$\int_0^2 \frac{x}{2}\,dx = A_2 = \frac{1}{2}(2)(1) = 1$$