Lecture 6

Thursday, January 19, 2023 8:30 AM

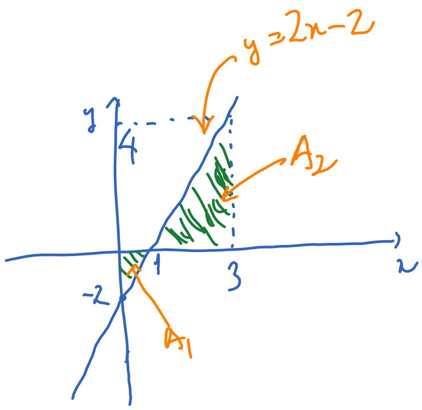
* Question

Algebraic area under curve: $\int_a^b f(x)\,dx$: definite integral of $f(x)$ from a to b.

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} L_n = \lim_{n\to\infty} R_n = \lim_{n\to\infty} M_n = \lim_{n\to\infty} T_n .$$

We will learn (next time) how to evaluate this integral without taking the limit or using geometry.

Ex

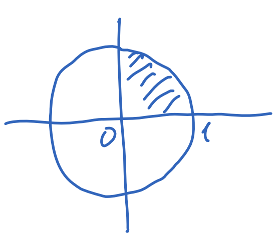


$$\int_0^3 (2x-2)\,dx = -A_1 + A_2$$

$$= -1 + \frac{1}{2}\times 4\times 2 = -1+4 = 3$$

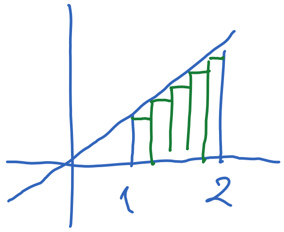
Ex

$$\int_0^1 \sqrt{1-x^2}\,dx = \frac{1}{4}\text{ area of circle} = \frac{1}{4}\pi$$



Ex

$$\int_1^2 x\,dx = ? \quad \text{(using limit)}$$



$$x_k = 1 + \frac{k}{n}$$

$$L_n = \sum_{k=0}^{n-1} f(x_k)\,\Delta x = \sum_{k=0}^{n-1} x_k \,\Delta x$$

$$\Delta x = \frac{2-1}{n} = \frac{1}{n}$$

$$= \sum_{k=0}^{n-1} \left(1 + \frac{k}{n}\right)\frac{1}{n}$$

$$= \frac{1}{n} \sum_{k=0}^{n-1} \left(1 + \frac{k}{n}\right)$$

$$= \frac{1}{n}\left(\sum_{k=0}^{n-1} 1 + \sum_{k=0}^{n-1} \frac{k}{n}\right)$$

$$= \frac{2}{n}\left(n + \frac{1}{n}\sum_{k=0}^{n-1} k\right) = \frac{2}{n}\left(n + \frac{1}{n}\,\frac{n(n-1)}{2}\right)$$

$$= \frac{1}{n}\left(n + \frac{n-1}{2}\right) = \frac{3n-1}{2n} \longrightarrow \frac{3}{2} \quad \text{as } n \to \infty$$

Therefore, $\displaystyle\int_1^2 x\,dx = \lim_{n\to\infty} L_n = \frac{3}{2}$