Lecture 7

Friday, January 20, 2023 10:25 AM

* Questions...

* Recall:

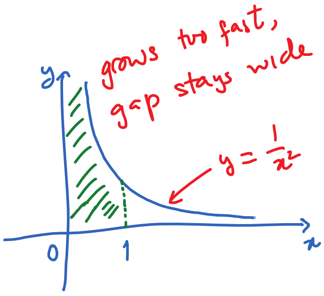
$$\int_a^b f(x)\,dx = \text{algebraic area under the graph of } f$$

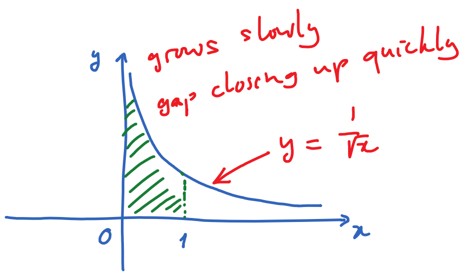
$$= \lim_{n\to\infty} L_n = \lim_{n\to\infty} R_n = \lim_{n\to\infty} M_n = \lim_{n\to\infty} T_n$$

<u>Definition</u>

If $\int_a^b f(x)dx$ exists (as a number) then f is said to be <u>integrable over $[a,b]$</u>.

Not all all functions are integrable!





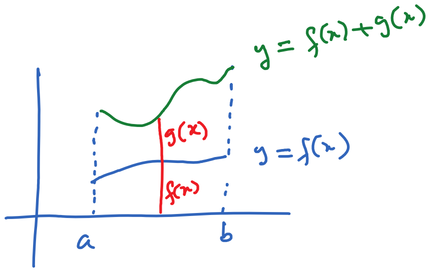
$$\int_0^1 \frac{1}{x^2}\,dx = \infty \qquad\qquad \int_0^1 \frac{1}{\sqrt{x}}\,dx = 2$$

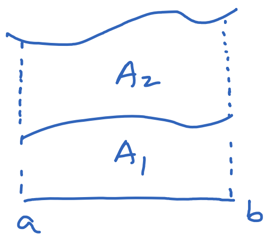
we'll learn later

why we get these values

* Properties of definite integrals

1) $$\int_a^b [f(x)+g(x)]\,dx = \int_a^b f(x)dx + \int_a^b g(x)\,dx$$

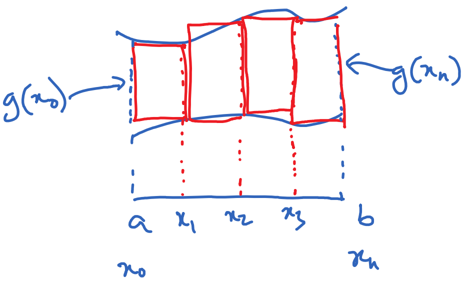




$$\int_a^b [f(x)+g(x)]dx = A_1 + A_2$$

$$A_1 = \int_a^b f(x)dx$$

$$A_2 \overset{?}{=} \int_a^b g(x)\,dx$$



$$A_2 \approx \sum g(x_k)\,\Delta x = R_n \approx \int_a^b g(x)\,dx$$

when $n \to \infty$,

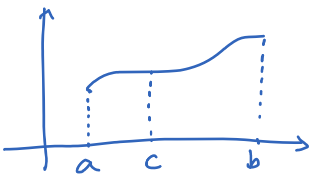
$$A_2 = \int_a^b g(x)\,dx$$

2) $$\int_a^a f(x)\,dx = 0$$

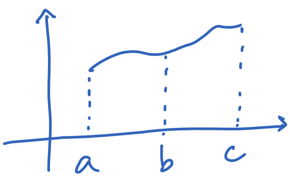
3) $$\int_a^b c f(x)\,dx = c \int_a^b f(x)\,dx$$

4) Convention: $$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

5) $$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$



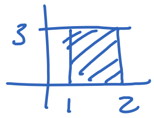
$$\int_a^b = \int_a^c + \int_c^b$$



$$\int_a^b = \int_a^c - \int_b^c = \int_a^c + \int_c^b$$

Ex $$\int_2^1 (2x-3)\,dx = \int_2^1 2x\,dx - \int_2^1 3\,dx = -\int_1^2 2x\,dx + \int_1^2 3\,dx$$

$$= -2\underbrace{\int_1^2 x\,dx}_{=\frac{3}{2}\text{ (last lecture)}} + \underbrace{\int_1^2 3\,dx}_{=3}$$



$$= 0$$