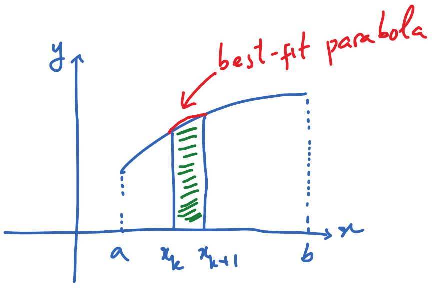
Lecture 8

Monday, January 23, 2023 10:58 AM

* Questions ...

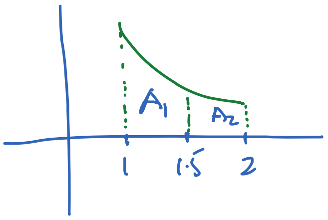
Simpson rule



$$\int_a^b f(x)\,dx = \sum_{k=0}^{n-1} \text{slat from } x_k \text{ to } x_{k+1}$$

$$\approx \underbrace{\sum_{k=0}^{n-1} \frac{1}{6}\left(f(x_k) + 4f\left(\frac{x_k + x_{k+1}}{2}\right) + f(x_{k+1})\right)\Delta x}_{\text{Simpson's rule}}$$

Ex $\int_1^2 \frac{1}{x}\,dx$ using Simpson's rule with $n=2$



$$A_1 \approx \frac{1}{6}\left(f(1) + 4f(1.25) + f(1.5)\right)\Delta x$$

$$= \frac{1}{6}\left(\frac{1}{1} + \frac{4}{1.25} + \frac{1}{1.5}\right)0.5 \approx 0.405$$

$$A_2 \approx \frac{1}{6}\left(f(1.5) + 4f(1.75) + f(2)\right)\Delta x$$

$$= \frac{1}{6}\left(\frac{1}{1.5} + \frac{4}{1.75} + \frac{1}{2}\right)0.5$$

$$\approx 0.2875$$

Thus, $\int_1^2 \frac{1}{x}\,dx \approx A_1 + A_2 \approx 0.6925$

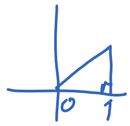
$\int_a^b f(x)\,dx$ can be computed geometrically (sometimes) and analytically using Riemann sums (more computationally involved).

There is another method to evaluate this integral. This method is algebraic in nature.

Let $S(x) = \int_a^x f(t)\,dt$.

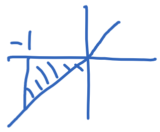
<u>Ex</u> $S(x) = \int_0^x t\,dt$

$S(0) = \int_0^0 t\,dt = 0$



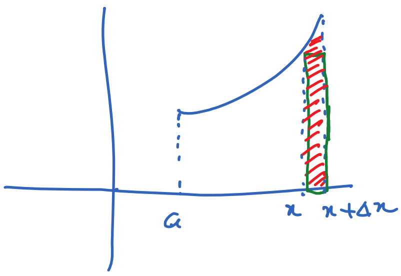
$S(1) = \int_0^1 t\,dt = \frac{1}{2}(1)(1) = \frac{1}{2}$

$S(2) = \int_0^2 t\,dt = \frac{1}{2}(2)(2) = 2$



$S(-1) = \int_0^{-1} = -\int_{-1}^0 t\,dt$

$= -\left(-\frac{1}{2}\right) = \frac{1}{2}$



$\underbrace{S(x+\Delta x) - S(x)} = \text{red area} \approx f(x)\Delta x$

$\approx S'(x)\Delta x$

$\boxed{S'(x) = f(x)}$ Thus, $S(x)$ is an antiderivative of $f(x)$.

$\int_a^b f(x)\,dx = S(b) = S(b) - \underbrace{S(a)}_{0}$