Final exam: Some problems for review

The exam will be taken at the regular classroom (Badgley Hall 146) on Thursday, March 23, from 1 PM to 3 PM. The material covered is Section 11.6 - 13.2, excluding 11.8 and 12.4. A 4" x 6" handwritten double-sided note card is allowed. A scientific calculator is allowed. Graphing/programmable/transmittable calculators are not allowed.

You should review the homework problems, quizzes, examples given in the textbook and in the lectures. It is always a good idea to study for the exam with someone. The types of problems you may be asked on the exam include:

- Compute directional derivatives.
- Find critical points of a function and classify them (local max, local min, saddle point, inclusive) using the Second Derivative Test.
- Find min/max of a function in a given region.
- Compute double integrals over a rectangle or a general region.
- Compute triple integrals over a rectangular box or a general solid.
- Find double/triple integral using a change of variables: polar/cylindrical/spherical coordinates.
- Sketch by hand a vector field.
- Find the line integral of a scalar function or a vector field over a curve.

Additional problems to practice:

1) Find $\int_C (x^2+y^2+z^2)ds$ where the curve C has a parametrization $x = t$, $y = \cos(2t)$, $z = \sin(2t)$, $0 \leq t \leq 2\pi$.

2) Find $\int_C F \cdot dr$ where $F(x, y) = (y, -x)$ and C is the straight line connecting $(1, 1)$ to $(-2, 2)$.

3) Find $\iint_R \frac{x-2y}{3x-y}\, dA$ where R is the parallelogram enclosed by the lines $x - 2y = 0$, $x - 2y = 4$, $3x - y = 1$, and $3x - y = 8$.

4) Find the integral $\iint_D y dA$ where D is the region bounded by $y = x - 2$ and $x = y^2$.

5) Find $\iiint_E z\, dV$ where E is the solid bounded by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0$, $y = 2x$, and $z = 0$ in the first octant.

6) Find all critical points of the function $f(x, y) = e^y(y^2 - x^2)$ and classify them using the Second Derivative Test.

7) Find the absolute maximum and minimum values of the function $f(x, y) = 4x + 6y - x^2 - y^2$ in the region $D = \{(x, y)|\ 0 \leq x \leq 4,\ 0 \leq y \leq 5\}$.

Solution keys:

1) $\sqrt{5}\left(\frac{8\pi^3}{3} + 2\pi\right)$

2) -4

3) $\frac{8\ln 8}{5}$

4) $9/4$

5) $81/16$

6) f has two critical points: $(0,0)$ and $(0,-2)$.
$(0,0)$ is a saddle point. $(0,-2)$ is where f attains a local maximum.

7) $\max_D f = 13$, attained at $(2,3)$
$\min_D f = 0$, attained at $(0,0)$ and $(4,0)$