

Lecture 1

Monday, January 9, 2023 8:21 AM

In Calc I, II, III, you learned functions of one variable.



With these tools, you can find length of a curve, surface area of a solid of revolution, volume of a solid of revolution,

3D objects are more general than that. They are diverse in geometry and require further mathematical tools to study.



In this course, we will study functions of more than one variable.

$$f(x, y, z, \dots)$$

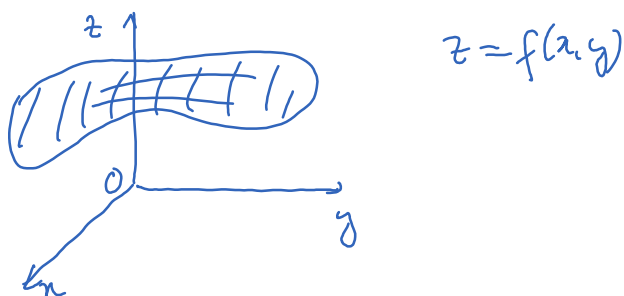
$$\underline{\underline{Ex}} \quad f(x, y) = x^2 + y^2$$

$$f(x, y, z) = xy + z$$

$$f(x, y, z, t) = x + te^{yz}$$

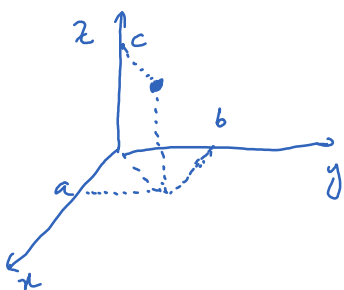
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The graph of a function $f(x,y)$ is a surface in 3D.



It is reasonable that we should understand the 3D rectangular coordinate system.

$P(a,b,c)$: a point is represented by a triple.

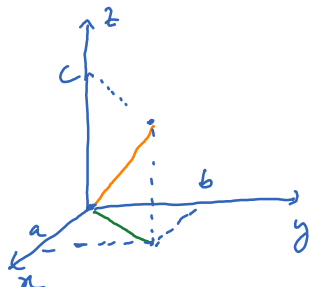
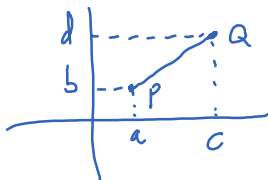


a = how far the point is to the yz plane
 b = " " " " xz plane
 c = " " " " xy plane

Ex sketch points $(0,1,2)$, $(1,0,2)$, $(0,1,2)$, $(1,1,1)$.

Distance

$$PA = \sqrt{(a-c)^2 + (b-d)^2}$$



green bar = $\sqrt{a^2 + b^2}$
 red bar = $\sqrt{\text{green}^2 + c^2}$
 $= \sqrt{a^2 + b^2 + c^2}$

Distance between two points $P(a, b, c)$ and $Q(d, e, f)$ is

$$PQ = \sqrt{(a-d)^2 + (b-e)^2 + (c-f)^2}$$

Ex the equation of the sphere centered at $(1, -1, 1)$ with radius 2 is

$$\sqrt{(x-1)^2 + (y+1)^2 + (z-1)^2} = 2$$

or equivalently,

$$(x-1)^2 + (y+1)^2 + (z-1)^2 = 2^2 = 4$$

Plot on Mathematica using `ContourPlot3D`.

Ex find the center and radius of the sphere given by eq:

$$x^2 + y^2 + z^2 + 2x + 4y - 2z = 5$$