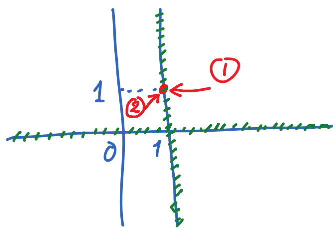
Lecture 11

Friday, January 27, 2023 10:58 AM

* Questions..

Limit of a multivariable function

$$\lim_{(x,y)\to(1,1)} \frac{x+y-2}{xy-y}$$



domain is the plane
except the line $y=1$
and the line $x=1$

On path ①. $y=1$, $x\to 1$

$$\frac{x+y-2}{xy-y} = \frac{x-1}{x-1} = 1 \to 1$$

On path ②. $x=y\to 1$

$$\frac{x+y-2}{xy-y} = \frac{2x-2}{x^2-x} = \frac{2(x-1)}{x(x-1)} = \frac{2}{x} \to 2$$

Because the limit of $\frac{x+y-2}{xy-y}$ as $(x,y)\to(1,1)$ on two different paths are different, $\lim_{(x,y)\to(1,1)} \frac{x+y-2}{xy-y}$ doesn't exist.

Ex $$\lim_{(x,y)\to(0,0)} \frac{x^3}{x^2+y^2}$$

$$-|x| \le x\frac{x^2}{x^2+y^2} \le |x|$$

$$-|x| \xrightarrow{x\to 0} 0 \xleftarrow{x\to 0} |x|$$

By the Squeeze theorem, $\lim_{(x,y)\to(0,0)} \frac{x^3}{x^2+y^2} = 0$.

Challenge

$$\lim_{(x,y)\to(0,0)} \frac{x^3}{x^2-y^2} = ? \qquad [\text{3 bonus points on your HW}]$$

Continuity

- If $\displaystyle\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$ then f is said to be continuous at (a,b).
- If f is continuous at any points (a,b) in a region R then f is said to be continuous on R.

Derivatives

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\frac{\partial}{\partial x} f(x,y) = \lim_{h\to 0} \frac{f(x+h,y)-f(x,y)}{h} \qquad (\text{also denoted as } f_x)$$

$$\frac{\partial}{\partial y} f(x,y) = \lim_{h\to 0} \frac{f(x,y+h)-f(x,y)}{h} \qquad (\text{also denoted as } f_y)$$

Ex $\quad f(x,y) = x^y$

Find f_x and f_y.