# Lecture 12

Monday, January 30, 2023 3:23 PM

* Question.

10.7 Prob 1 domain of $r(t) = (\sqrt{4-t^2}, e^{-3t}, \ln(t+1))$

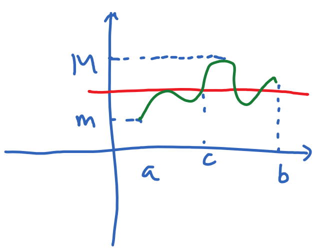
$D = \{t : 4-t^2 \geq 0 \text{ and } t+1 > 0\}$

$= \{t : -2 \leq t \leq 2 \text{ and } t > -1\}$

$= (-1, 2]$

Recall the topic progression of Calc I.

limit $\longrightarrow$ continuous functions $\longrightarrow$ Intermediate value theorem $\longrightarrow$ Derivatives




Let $f$ be continuous on $[a,b]$ and let

$m = \min_{[a,b]} f$, $M = \max_{[a,b]} f$

Then for any $y \in [m, M]$, there exists $c \in [a,b]$ such that $f(c) = y$.

* Intermediate value theorem for multivariable calculus:

Let $f = f(x,y)$ be a continuous function on a connected region $D$.

Let $m = \min_D f$ and $M = \max_D f$. Then for any $z \in [m, M]$, there exists $(x,y) \in D$ such that $f(x,y) = z$.

In Calc I, we learned derivatives to deal with problems related to rate of change, tangent line, shape of a graph, minimizing / maximizing a function.

For similar purposes, we will learn derivatives of a multivariable function.

## Derivatives

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{\partial}{\partial x} f(x,y) = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$ (also denoted as $f_x$)

$$\frac{\partial}{\partial y} f(x,y) = \lim_{h \to 0} \frac{f(x,y+h) - f(x,y)}{h}$$ (also denoted as $f_y$)

**Ex** $f(x,y) = x^y$

Find $f_x$ and $f_y$.

## Higher derivatives

$f_{xx} = (f_x)_x$, $f_{xy} = (f_x)_y$, $f_{yy} = (f_y)_y$

also

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right), \quad \frac{\partial^2 f}{\partial y \partial x}, \quad \frac{\partial^2 f}{\partial y^2}$$

**Ex** $f(x,y,z) = x^2 y + y^2 z + z^2 x$

$f_x(x,y,z) = 2xy + z^2$

$$\left.\begin{array}{l} f_{xy}(x,y,z) = 2x \\ f_{yx} = (x^2 + 2yz)_{,x} = 2x \end{array}\right\} f_{xy} = f_{yx}$$