Lecture 19

Monday, February 13, 2023 12:38 PM

Questions ...

* Directional derivatives

$$D_u f(x,y) = \underbrace{\nabla f(x,y)}_{\text{gradient vector}} \cdot u$$

$$D_u f(x,y,z) = \nabla f(x,y,z) \cdot u$$

$D_u f(x,y,z) \to \max$ if $u = \frac{\nabla f}{|\nabla f|}$

$D_u f(x,y,z) \to \min$ if $u = -\frac{\nabla f}{|\nabla f|}$

$D_u f(x,y,z) = 0$ if $u \perp \nabla f$

Ex $f(x,y) = x^2 + 2y^2$ at $(x_0, y_0) = (1,1)$

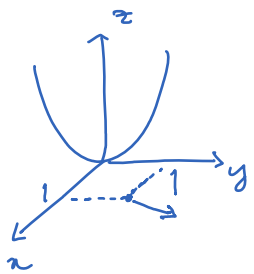
Find the direction that corresponds to the fastest increase of f.

$f_x = 2x \rightsquigarrow f_x(1,1) = 2$

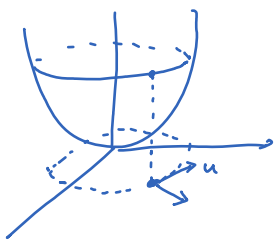
$f_y = 4y \rightsquigarrow f_y(1,1) = 4$

$\nabla f(1,1) = (2,4)$

The direction is $u = \frac{(2,4)}{|(2,4)|} = \frac{(2,4)}{\sqrt{20}} = \frac{(1,2)}{\sqrt{5}} = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$.



Gradient is always perpendicular to level set.



Use Mathematica to visualize. ContourPlot and VectorPlot

Application Write the equation of the tangent plane to the surface $x^2+y^2+4z^2=1$ at the point $(\frac{1}{3}, \frac{2}{3}, \frac{1}{3})$.

Let

$$f(x,y,z) = x^2+y^2+4z^2$$

The surface is the 1-level set of the function f, which is perpendicular to vector $\nabla f(\frac{1}{3}, \frac{2}{3}, \frac{1}{3})$.

$$\nabla f = (2x, 2y, 8z)$$

$$\nabla f(\tfrac{1}{3}, \tfrac{2}{3}, \tfrac{1}{3}) = (\tfrac{2}{3}, \tfrac{4}{3}, \tfrac{8}{3}) = \tfrac{2}{3}\underbrace{(1,2,4)}_{\text{normal vector}}$$

Equation of tangent plane is

$$1(x-\tfrac{1}{3}) + 2(y-\tfrac{2}{3}) + 4(z-\tfrac{1}{3}) = 0$$

or equivalently,

$$x+2y+4z-3=0$$