Lecture 20

Tuesday, February 14, 2023 12:08 PM

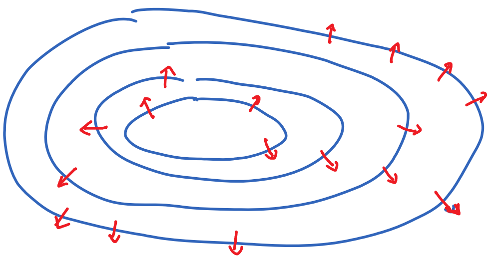
* Question

Visualize the gradient vector field and the level sets.

$$f(x,y) = x^2 + 2y^2$$

ContourPlot[x^2+2y^2 , {x, -2, 2}, {y, -2, 2}]

VectorPlot[{2x, 4y}, {x, -2, 2}, {y, -2, 2}]



The lighter color you see on Mathematica is where f has higher values.

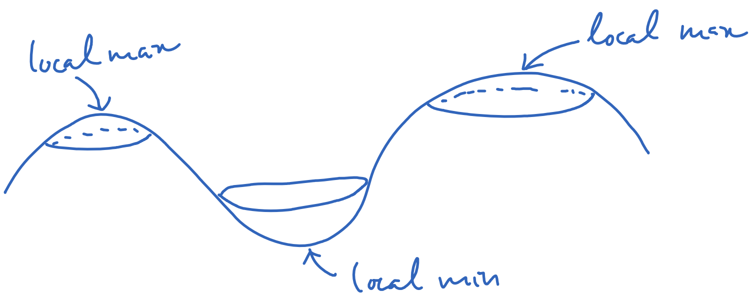
Recall that the gradient vectors point to the direction where f increases the most.

<u>Principle</u> gradient vectors are perpendicular to the level sets.

This principle can be used to find the tangent plane of a surface. See the lecture note of the previous lecture.

<u>Optimization problem</u>

$$f(x,y) \to \min, \max \text{ on region } D.$$



At the peaks and valleys, the tangent planes are parallel to the xy-plane.

$$z = z_0 + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

Normal vector $(\underbrace{f_x, f_y}, -1)$

has to be $0, 0$ so that the tangent plane is parallel to the xy-plane.

Thus, $\nabla f(x_0, y_0) = 0$.

<u>Def</u> If $\nabla f(x_0, y_0) = (0, 0)$, the point (x_0, y_0) is called a <u>critical point</u> of f.