Lecture 24

Tuesday, February 21, 2023 12:23 PM

* Questions ....

Double integral

$$\iint_{[a,b]\times[c,d]} f(x,y)\,dA = \lim_{m,n\to\infty} \sum_{j=0}^{m-1} \sum_{i=0}^{n-1} f(x_i^*, y_j^*)\,\Delta A$$

is the (exact) area under the surface and above the rectangle

$R = [a,b] \times [c,d]$.

Finding the limit is tricky, even with simple functions $f(x,y)$.

We need a better way

In Calc I

$$\int_a^b f(x)\,dx = F(b) - F(a), \text{ where } F \text{ is an antiderivative of } f.$$

We need something like this for multivariable functions.

$$\sum_{j=0}^{m-1} \boxed{\sum_{i=0}^{n-1} f(x_i^*, y_j^*)\,\Delta x}\,\Delta y \approx \sum_{j=0}^{m-1} \left(\int_a^b f(x, y_j^*)\,dx \right) \Delta y$$

$$\approx \int_a^b f(x, y_j^*)\,dx \qquad \approx \int_c^d \int_a^b f(x,y)\,dx\,dy$$

Theorem:

$$\iint_{[a,b]\times[c,d]} f(x,y)\,dA = \int_c^d \int_a^b f(x,y)\,dx\,dy = \int_a^b \int_c^d f(x,y)\,dy\,dx$$

Ex Find $\displaystyle\iint_{[-1,2]\times[0,1]} (x^2+y^2)\,dA$

$$\iint_{[-1,2]\times[0,1]} (x^2+y^2)\,dA = \int_{-1}^{2}\int_{0}^{1}(x^2+y^2)\,dy\,dx$$

$$= \int_{-1}^{2}\left(x^2y+\frac{y^3}{3}\right)\Big|_{y=0}^{y=1}\,dx$$

$$= \int_{-1}^{2}\left(x^2+\frac{1}{3}\right)dx = \left(\frac{x^3}{3}+\frac{x}{3}\right)\Big|_{-1}^{2} = \frac{10}{3}$$